Average decay of the Fourier transform and applications

ABSTRACT: Let K be a convex body in $\mathbf{R}^d$. It is well known that if ∂K is smooth and has everywhere non-vanishing curvature, then

$$|\widehat{\chi}_K(R\omega)| \ll R^{-\frac{d+1}{2}}. \tag{1}$$

If the Gaussian curvature on ∂K is allowed to vanish, and/or smoothness is taken away, the estimate (1) does not in general hold. However, for many problems in harmonic analysis, analytic number theory, and other areas, it is important to know whether the estimate (1) holds on average, for example whether

$$\int_{S^{d-1}} |\widehat{\chi}_K(R\omega)|^2 d\omega \ll R^{-(d+1)}. \tag{2}$$

We shall see that the estimate (2) always holds. Moreover, we shall see that it holds for a very general class domains, which leads to some surprising conclusions about the distribution of lattice points in ugly domains and related problems.