**Harmonic Analysis techniques in Several Complex Variables**

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**Abstract.** This talk concerns the application of relatively classical tools from real harmonic analysis (namely, the $T(1)$-theorem for spaces of homogenous type) to the novel context of several complex variables. Specifically, I will present recent joint work with E. M. Stein (Princeton U.) on the extension to higher dimension of Calderón's and Coifman-McIntosh-Meyer's seminal results about the Cauchy integral for a Lipschitz planar curve (interpreted as the boundary of a Lipschitz domain $D \subset \mathbb{C}$). From the point of view of complex analysis, a fundamental feature of the 1-dimensional Cauchy kernel:

$$H(w, z) = \frac{1}{2\pi i} \frac{dw}{w - z}$$

is that it is holomorphic (that is, analytic) as a function of $z \in D$. In great contrast with the one-dimensional theory, in higher dimension there is no obvious holomorphic analogue of $H(w, z)$. This is because of geometric obstructions (the Levi problem), which in dimension 1 are irrelevant.

A good candidate kernel for the higher dimensional setting was first identified by Jean Leray in the context of a $C^\infty$-smooth, convex domain $D$: while these conditions on $D$ can be relaxed a bit, if the domain is less than $C^2$-smooth (never mind Lipschitz!) Leray's construction becomes conceptually problematic.

In this talk I will present *(a)*, the construction of the Cauchy-Leray kernel and *(b)*, the $L^p(bD)$-boundedness of the induced singular integral operator under the weakest currently known assumptions on the domain's regularity – in the case of a planar domain these are akin to Lipschitz boundary, but in our higher-dimensional context the assumptions we make are in fact optimal. The proofs rely in a fundamental way on a suitably adapted version of the so-called "$T(1)$-theorem technique" from real harmonic analysis.

Time permitting, I will describe applications of this work to complex function theory – specifically, to the Szegő and Bergman projections (that is, the orthogonal projections of $L^2$ onto, respectively, the Hardy and Bergman spaces of holomorphic functions).

## References

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[LS-2] Lanzani L. and Stein E. M., *The Cauchy Integral in $\mathbb{C}^n$ for domains with minimal smoothness*, Adv. Math. **264** (2014) 776 – 830.

[LS-3] Lanzani L. and Stein E. M., *The Cauchy-Leray Integral: counter-examples to the $L^p$-theory*, Indiana Math. J., to appear.