Bounded variation, convexity, and almost-orthogonality

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Abstract.

Let $N \geq 2$ be fixed. Suppose that, for every dyadic cube Q in $\mathbf{R}^d$, we have: N convex regions $\{R_i(Q)\}_1^N$, subsets of Q; and N complex numbers $\{c_i(Q)\}_1^N$ such that $|c_i(Q)| \leq 1$ and $\sum_1^N c_i(Q)|R_i(Q)| = 0$. Define $\tilde{h}_{(Q)}(x) \equiv |Q|^{-1/2}(\sum_1^N c_i(Q)\chi_{R_i(Q)}(x))$. We prove that there is an absolute constant C, independent of N and d, so that, for all such collections $\{\tilde{h}_{(Q)}\}_Q$ and all finite linear combinations $\sum \lambda_Q \tilde{h}_{(Q)}(x)$,

$$\left\|\sum \lambda_Q \tilde{h}_{(Q)}\right\|_2 \leq CNd\left(\sum |\lambda_Q|^2\right)^{1/2}.$$

Our result is a special case of a technical theorem, which we prove.