ON MEASURE THEORETIC PROJECTION BODIES

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Abstract

The inequalities of Petty and Zhang are affine isoperimetric-type inequalities providing sharp bounds for $\mathrm{vol}_n^{n-1}(K)\mathrm{vol}_n(\Pi^\circ K)$, where ΠK is a projection body of a convex body K is the convex body with support function given by

$$h_{\Pi K}(\theta) = \mathrm{vol}_{n-1}(K|\theta^\perp), \quad \theta \in \mathbb{S}^{n-1},$$

where $\theta^\perp$ denotes the hyperplane orthogonal to the direction θ. The upper bound, due to Petty, and referred to as Petty's projection inequality attains equality only when K is an ellipsoid, and the lower bound is due to Zhang and equality occurs only when K is a simplex.

In this talk, we present a number of generalizations of Zhang's inequality to the setting of measures.

In addition, we introduce extensions of the projection body operator Π to the setting of arbitrary measures, that is, given a measure μ on $\mathbb{R}^n$ with continuous density φ, $\Pi_\mu K$ is the convex bodies whose support function is given by

$$h_{\Pi_\mu K}(\theta) = \frac{1}{2}\int_{\partial K} |\langle \theta, n_K(y)\rangle| \phi(y) dy,$$

where ∂K denotes the boundary of K and $n_K(y)$ denotes the outer unit normal of ∂K at y. We remark that the support function $h_{\pi_\mu K}$ has been deeply studied in the literature, and is an example of a generalized zoniod when φ is taken to be even.

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