Boundary Value Problems for Higher Order Elliptic Operators

As is well known, many phenomena in Engineering and Mathematical Physics can be modeled by means of boundary value problems for a certain elliptic differential operator $\mathcal{L}$ in a domain Ω.

When $\mathcal{L}$ is a differential operator of second order, a variety of tools are available for dealing with such problems including boundary integral methods, variational methods, harmonic measure techniques, and methods based on classical harmonic analysis. The situation when the differential operator has higher order (as is the case for instance with anisotropic plate bending when one deals with fourth order) stands in sharp contrast with this as only fewer option could be successfully implemented. Alberto Calderón, one of the founders of the modern theory of Singular Integral Operators, has advocated in the seventies the use of layer potentials for the treatment of higher order elliptic boundary value problems. While the layer potential method has proved to be tremendously successful in the treatment of second order problems, this approach is currently insufficiently developed to deal with the intricacies of the theory of higher order operators. In fact, it is largely absent from the literature dealing with such problems.

In this talk I will discuss recent progress in developing a multiple layer potential approach for the treatment of boundary value problems associated with higher order elliptic differential operators. This is done in a very general class of domains which is in the nature of best possible from the point of view of geometric measure theory.