Solyanik Estimates in Harmonic Analysis

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Abstract. Let \mathcal{B} be a collection of open sets in \mathbb{R}^n . Associated to \mathcal{B} is the geometric maximal operator $M_{\mathcal{B}}$ defined by

$$M_{\mathcal{B}}f(x) = \sup_{x \in R \in \mathcal{B}} \int_{R} |f| .$$

For $0 < \alpha < 1$, the associated Tauberian constant $C_{\mathcal{B}}(\alpha)$ is given by

$$C_{\mathcal{B}}(\alpha) = \sup_{E \subset \mathbb{R}^n : 0 < |E| < \infty} \frac{1}{|E|} |\{x \in \mathbb{R}^n : M_{\mathcal{B}}\chi_E(x) > \alpha\}|.$$

A maximal operator $M_{\mathcal{B}}$ such that $\lim_{\alpha \to 1^{-}} C_{\mathcal{B}}(\alpha) = 1$ is said to satisfy a *Solyanik estimate*.

In this talk we will prove that the uncentered Hardy-Littlewood maximal operator satisfies a Solyanik estimate. Moreover, we will indicate applications of Solyanik estimates to smoothness properties of Tauberian constants and to weighted norm inequalities. We will also discuss several fascinating open problems regarding Solyanik estimates. This research is joint with Ioannis Parissis.