# Pseudospectral approximation of eigenvalues of derivative operators with non-local boundary conditions

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**Abstract**

By taking as "prototype problem" the following linear differential system with discrete delay

$$\begin{cases} x'(t) = L_0x(t) + L_1x(t-\tau), & t \geq 0 \\ x(t) = \phi(t), & -\tau \leq t \leq 0 \end{cases} \tag{1}$$

where $L_0, L_1 \in \mathbb{C}^{m\times m}$ and $\tau > 0$, we will present the reformulation of a retarded functional differential equation as the abstract Cauchy problem on $X = C\left([-\tau, 0], \mathbb{C}^m\right)$

$$\begin{cases} y'(t) = \mathcal{A}y(t), & t \geq 0 \\ y(0) = \phi \end{cases}$$

where $y : [0, +\infty) \to D(\mathcal{A}) \subseteq X$, $\phi \in D(\mathcal{A})$ and $\mathcal{A} : D(\mathcal{A}) \to X$ is a derivative operator which satisfies suitable boundary conditions given by the particular system (1) considered and contained in the domain $D(\mathcal{A})$. The asymptotic behavior of the solutions of (1) is completely described by the eigenvalues of the operator $\mathcal{A}$. In particular the zero solution of (1) is asymptotically stable if and only if all the eigenvalues of $\mathcal{A}$ have strictly negative real part.

The analysis of the spectrum of derivative operators with general non-local boundary conditions represents an important tool for the investigation of the stability properties of solutions for more general classes of equations, which are important for applications: linear autonomous differential systems with multiple discrete and distributed delays, equations modeling age-structured population dynamics, reaction diffusion equations with delay in the reaction term, equations of mixed type (i.e. advanced-retarded equations), neutral delay equations.

It is thus relevant to have a numerical technique to approximate the eigenvalues of derivative operators $\mathcal{A}$ under non-local boundary conditions. We propose to discretize the operators $\mathcal{A}$ by pseudospectral methods and turn the original problem into a suitable matrix eigenvalue problem. This approch is particular efficient due to the the well-known "spectral accuracy" convergence of pseudospectral methods.