Applied Mathematics

McGill Applied Mathematics Seminar Series

November 25, 2002

Dr. Adrian Vetta

School of Computer Science, McGill

Title: "On the quality of competitive equilibria in games with submodular social objective functions."

Monday November 25, 2002 Time: 2:35 pm Room: Burnside 1205

Coffee and refreshments will be served after the seminar

Abstract: The costs incurred by society due to a *lack of complete information* or a *lack of unbounded computational resources* have been examined in depth. For example, the fields of on-line algorithms and approximation algorithms were developed in response to these two problems. In such analyses, however, the assumption is made that there is a single authority (or agent) whose goal is to optimise some objective function. What happens when there is a clear social objective function but no single authority, i.e. what if there are many agents whose goals are to optimise their own private objective functions, rather than to collectively optimise the social objective function? We quantify the cost to society of the resultant *lack of coordination/cooperation* for a broad class of games: the value of an outcome to a society is measured via a submodular utility function (submodularity has a natural economic interpretation: decreasing marginal utility) but decisions are made by non-cooperative agents who seek to maximise their own private utility. In particular, we show that for non-decreasing, submodular social utility functions, any Nash equilibrium gives an expected social utility within a factor $1+\delta$ of optimal, where $0 \leq \delta \leq 1$ is a number based upon the discrete curvature of the function. Examples of problems that fall into this framework include competitive facility location, traffic routing in networks, and combinatorial auctions.

We show how to generalise our results to the case in which the private agents are also restricted by bounded computational resources, and present a simple condition under which all sets of social and private utility functions have the desirable property that they induce pure strategy Nash equilibria.

Figure 1: A Nash equilibrium?