Formulas

Frenet formulas: $\mathbf{t}' = k\mathbf{n}, \mathbf{n}' = -k\mathbf{t} - \tau\mathbf{b}, \mathbf{b}' = \tau\mathbf{n}$.

Rodrigues formula for line of curvature: $N'(t) = \lambda(t)\alpha'(t)$.

An **asymptotic direction** of a surface S at a point p is a direction of $T_p(S)$ for which the normal curvature is 0.

The Second Fundamental form:

$$
\begin{aligned}
e &= -< Nu, \mathbf{x}_u > = < N, \mathbf{x}_{uu} >, \\
f &= -< N_u, \mathbf{x}_v > = -< N_v, \mathbf{x}_u > = < N, \mathbf{x}_{uv} >, \\
g &= -< N_v, \mathbf{x}_v > = < N, \mathbf{x}_{vv} > .
\end{aligned}
$$

Weingarten equations:

$$
\begin{aligned}
a_{11} &= \frac{fF - eG}{EG - F^2}, \quad a_{12} = \frac{gF - fG}{EG - F^2}, \\
a_{21} &= \frac{eF - fE}{EG - F^2}, \quad a_{22} = \frac{fF - gE}{EG - F^2}.
\end{aligned}
$$

Gauss curvature: $K = \frac{eg - f^2}{EG - F^2}$, **mean curvature**: $H = \frac{1}{2}\frac{eG - 2fF + gE}{EG - F^2}$.

Formulas for the Christoffel symbols: $\Gamma_{ij}^k = \frac{1}{2}\sum_{l=1}^2 g^{kl}(\frac{\partial g_{il}}{\partial x_j} + \frac{\partial g_{jl}}{\partial x_i} - \frac{\partial g_{ij}}{\partial x_l})$, where $u = x_1, v = x_2$, $g_{ij} = < X_i, X_j >$, and (g^{ij}) is the inverse matrix of (g_{ij}).

Gauss equation for orthogonal parametrization (i.e, $F \equiv 0$):

$$
K = -\frac{1}{2\sqrt{EG}}\{(\frac{E_v}{\sqrt{EG}})_v + (\frac{G_u}{\sqrt{EG}})_u\}.
$$

Gauss curvature in geodesic coordinates: $K = -\frac{(\sqrt{G})_{\rho\rho}}{\sqrt{G}}$.