P 174. #24 (a). After a rotation and a translation, we may express S as a graph over $\mathbb{R}^2_{(x,y)}$, where $p=(0,0)$, $\mathbb{R}^2_{(x,y)}$ is the tangent plane of S at p. (Why?). So, locally, $S = \{(x,y)\in\mathbb{R}^2 \mid z = f(x,y)\}$ and at $(0,0)$, $z=0$, and $\nabla f(0,0) = (0,0)$ (Since $\mathbb{R}^2_{(x,y)}$ is the tangent plane). So at p, the principal curvatures of S are the eigenvalues of $(\nabla^2 f(0,0))$. After one more rotation, we may assume $(\nabla^2 f(0,0)) = \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix}$. By the assumption, $k_1 k_2 > 0$. they have the same sign, by taylor expansion

$f(x,y) = \frac{1}{2}(k_1 x_1^2 + k_2 x_2^2) + O\left((x^2+y^2)^{3/2}\right)$, now it's clear either $f(x,y) \geq 0$ (near $(0,0)$) or $f(x,y) \leq 0$. and strict inequality if $(x,y) \neq (0,0)$.

(b). following the same argument as in (a)

$f(x,y) = \frac{1}{2}(k_1 x_1^2 + k_2 x_2^2) + O\left((x^2+y^2)^{3/2}\right)$

If S is locally convex, either $f(x,y) \geq 0$ for $\forall (x,y)$ close to $(0,0)$, or $f(x,y) \leq 0$ $\forall (x,y)$ close to $(0,0)$. Now, we can deduce k_1, k_2 must have the same sign (or one of them is 0).

P212. #13 (a). Since S is without umbilical point. $\forall p \in S$ we may find a local orthogonal parametrization such that $\vec{e}_1$ and $\vec{e}_2$ are the directions of line of curvatures, i.e. $dN_q(\vec{e}_i) = k_i \vec{e}_i$, k_i the principal curvatures $(i=1,2)$. Now $\forall w_1, w_2 \in T_q S$, $dN_q(w_i) = a_i \vec{e}_1 + b_i \vec{e}_2$. so $\langle dN_q(\vec{w}_1), dN_q(\vec{w}_2)\rangle = a_1 a_2 k_1^2 + b_1 b_2 k_2^2$

$$= k_1^2 \langle \vec{w}_1, \vec{w}_2\rangle + (k_1^2 - k_2^2) b_1 b_2$$

Now, it is a simple algebra that

$k_1^2 \langle \vec{w}_1, \vec{w}_2\rangle + (k_1^2 - k_2^2) b_1 b_2 = \lambda \langle \vec{w}_1, \vec{w}_2\rangle$, $\forall a_i, b_i$ if and only if $k_1 = -k_2$, i.e. $H=0$.

(b). follows from (a).

P213. #14 , P.228 #4 direct computations.

P228. #18. as $E=\lambda^2\bar{E}$, $F=\lambda^2\bar{F}$, $G=\lambda^2\bar{G} \Rightarrow dA=\sqrt{GE-F^2}\,du\,dv$

$= \lambda^2\sqrt{\bar{G}\bar{E}-\bar{F}^2}\,du\,dv = \lambda^2\,d\bar{A} \Rightarrow \forall\,\delta>0.$

$$\int_{-\delta}^{\delta}\int_{-\delta}^{\delta}\lambda^2\sqrt{\bar{E}\bar{G}-\bar{F}^2}\,du\,dv = \int_{-\delta}^{\delta}\int_{-\delta}^{\delta}\sqrt{EG-F^2}\,du\,dv \overset{\text{by assumption}}{=} \int_{-\delta}^{\delta}\int_{-\delta}^{\delta}\sqrt{\bar{E}\bar{G}-\bar{F}^2}\,du\,dv$$

dividing by δ^2, $\delta\to 0$. $\Rightarrow$ $\lambda^2\sqrt{\bar{E}\bar{G}-\bar{F}^2}=\sqrt{\bar{E}\bar{G}-\bar{F}^2} \Rightarrow \lambda^2=1$

P237. #1. $\because F\equiv 0$, $\Gamma_{11}^1=\frac{1}{2}\frac{E_u}{E}$, $\Gamma_{11}^2=-\frac{1}{2}\frac{E_v}{G}$, $\Gamma_{12}^1=\frac{1}{2}\frac{E_v}{E}$, $\Gamma_{12}^2=\frac{1}{2}\frac{G_u}{G}$

$\Gamma_{11}^2=-\frac{1}{2}\frac{G_u}{E}$, $\Gamma_{22}^2=\frac{1}{2}\frac{G_v}{G}$. Put this to the Gauss formula

((5) in page 234)

#2. follows from #1.

P260. #2 $k^2=k_g^2+k_n^2$. $k\equiv 0 \Leftrightarrow k_g\equiv 0$, $k_n\equiv 0$.