MATH 248 ExaminationDecember 21, 2011

Problem 1 [8], Show that $A = \{(x, y) \in \mathbb{R}^2 | y = 1\}$ is a closed set in $\mathbb{R}^2$. What is the boundary of A?

Problem 2 [12], Set

$$f(x, y) = \begin{cases} \frac{x^3 y}{x^4+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0). \end{cases} \tag{1}$$

Show that f is a continuous function in $\mathbb{R}^2$. Determine all the points in $\mathbb{R}^2$ where f is differentiable.

Problem 3. [10] Find the absolute minimum and maximum for the function $f(x, y, z) = x^4 + y^4 + z^4$ on the set $\{(x, y, z) | x^2 + y^2 + z^2 \leq 1\}$.

Problem 4. [10] Let S be the surface defined by the graph $z = \frac{y^3}{\sqrt{3}} + \sqrt{2}x$ over $\Omega = \{x^{\frac{1}{3}} \leq y \leq 1, 0 \leq x \leq 1\}$. Find the area of the surface S.

Problem 5. [10] Find integral $\int\int_\Omega (x^4 - y^4) e^{xy} dA$, where Ω is the region in $\mathbb{R}^2$ in the first quadrant that is enclosed by the hyperbolas

$$x^2 - y^2 = 3, x^2 - y^2 = 4, xy = 1, xy = 3.$$

Problem 6. [10] Let Ω be an unbounded domain in $\mathbb{R}^2$, let $f(x, y), g(x, y)$ be two continuous functions defined in Ω. Suppose $0 \leq g(x, y) \leq f(x, y), \forall (x, y) \in \Omega$ and suppose $\int\int_\Omega g(x, y) dA$ does not exist, show that $\int\int_\Omega f(x, y) dA$ can not exist.

Problem 7. [10] Find the line integral $\int_{\mathbf{C}} \mathbf{F} \cdot d\mathbf{s}$, where $\mathbf{F} = \mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and $\mathbf{C}(\mathbf{t}) = (te^{(t-1)^5}, te^{(t-1)^6}, te^{(t-1)^7}), 0 \leq t \leq 1$ is an curve from the origin to the point $(1, 1, 1)$.

Problem 8. [10] Let C^+ be the curve in $\mathbb{R}^2$ defined by $x^4 + y^4 = 1$ with positive orientation. Evaluate $\int_{C^+} \frac{xdy - ydx}{x^2+y^2}$.

Problem 9. [10] Let $\mathbf{a} = (\mathbf{1}, \mathbf{0}, \mathbf{0})$ and let $\mathbf{v}$ be a fixed constant vectors on a surface S, prove that,

$$2 \int\int_S \mathbf{a} \cdot \mathbf{n} dS = \int_{\partial S} (\mathbf{a} \times (\mathbf{p} + \mathbf{v})) \cdot d\mathbf{s},$$

where $\mathbf{n}$ is the normal of S and $\mathbf{p}(x, y, z) =< x, y, z >$.

Problem 10. [10] Evaluate $\int\int_\Sigma \mathbf{F}\cdot\mathbf{n}dS$, where $\mathbf{F} = xz^2\mathbf{i} + z^3\mathbf{j} + z(x+y)\mathbf{k}$ and $\Sigma = \{\frac{x^2}{4} + \frac{y^2}{2} + z^2 = 1\}$.