

Fourth Assignment, due on October 31, 2014.

Problem 1 [10], Evaluate $\int \int_D e^{x-y} dx dy$, where D is the interior of the triangle with vertices $(0, 0)$, $(1, 3)$, and $(2, 2)$.

Problem 2 [10], Given that the double integral $\int \int_D f(x, y) dx dy$ of a positive continuous function f equals the iterated integral $\int_0^1 [\int_{x^2}^x f(x, y) dy] dx$, sketch the region D and interchange the order of integration.

Problem 3 [10], Find the volume of the region common to the intersecting cylinders $x^2 + y^2 \leq a^2$ and $x^2 + z^2 \leq a^2$.

Problem 4 [10],

- (a.) Sketch the region for the integral $\int_0^1 \int_0^x \int_0^y f(x, y, z) dz dy dx$.
- (b.) Write the integral with the integration order $dx dy dz$.

Problem 5 [10], For the region $W = \{(x, y, z) | |x| \leq 1, |y| \leq 1, z \geq 0 \text{ and } x^2 + y^2 + z^2 \leq 1\}$, find the appropriate limits $\phi_1(x)$, $\phi_2(x)$, $\gamma_1(x, y)$ and $\gamma_2(x, y)$, and write the triple integral over W as iterated integral in the form

$$\int \int \int_W f dV = \int_a^b \left\{ \int_{\phi_1(x)}^{\phi_2(x)} \left[\int_{\gamma_1(x, y)}^{\gamma_2(x, y)} f(x, y, z) dz \right] dy \right\} dx.$$

Problem 6 [10], Calculate $\int \int_R \frac{1}{x+y} dy dx$, where R is the region bounded by $x = 0$, $y = 0$, $x + y = 1$, $x + y = 4$, by using the mapping $T(u, v) = (u - uv, uv)$.

Problem 7 [10], Let D be the unit disk. Express $\int \int_D (1 + x^2 + y^2)^{\frac{3}{2}} dx dy$ as an integral over $[0, 1] \times [0, 2\pi]$ and evaluate.

Problem 8 [10], Use spherical coordinates to evaluate

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2-y^2}} \frac{\sqrt{x^2 + y^2 + z^2}}{1 + [x^2 + y^2 + z^2]^2} dz dy dx.$$

Problem 9 [10], Let f be continuous and let B_r be the ball of radius $r > 0$ centered at (x_0, y_0, z_0) . Let $V(B_r)$ be the volume of B_r . Prove that

$$\lim_{r \rightarrow 0} \frac{1}{V(B_r)} \int \int \int_{B_r} f(x, y, z) dV = f(x_0, y_0, z_0).$$

Problem 10 [10], Find improper integral $\int \int \int_{\mathbb{R}^3} f(x, y, z) dx dy dz$, where

$$f(x, y, z) = \frac{1}{[1 + (x^2 + y^2 + z^2)^{\frac{3}{2}}]^{\frac{3}{2}}}.$$