Fourth Assignment, due on November 2, 2015.

Problem 1 [10], Evaluate $\int\int_D e^{y-x}dxdy$, where D is the interior of the triangle with vertices $(0,0),(3,1)$, and $(2,2)$.

Problem 2 [10], Given that the double integral $\int\int_D f(x,y)dxdy$ of a positive continuous function f equals the iterated integral $\int_0^1[\int_{x^2}^x f(x,y)dy]dx$, sketch the region D and interchange the order of integration.

Problem 3 [10], Find the volume of the region common to the intersecting cylinders $x^2+z^2 \le a^2$ and $y^2+z^2 \le a^2$.

Problem 4 [10],

(a.) Sketch the region for the integral $\int_0^1\int_0^x\int_0^y f(x,y,z)dzdydx$.

(b.) Write the integral with the integration order $dxdydz$.

Problem 5 [10], For the region $W = \{(x,y,z)||x| \le 1, |y| \le 1, z \ge 0 \text{ and } x^2+y^2+z^2 \le 1\}$, find the appropriate limits $\phi_1(x), \phi_2(x), \gamma_1(x,y)$ and $\gamma_2(x,y)$, and write the triple integral over W as iterated integral in the form

$$\int\int\int_W f dV = \int_a^b \{\int_{\phi_1(x)}^{\phi_2(x)} [\int_{\gamma_1(x,y)}^{\gamma_2(x,y)} f(x,y,z)dz]dy\}dx.$$

Problem 6 [10], Calculate $\int\int_R \frac{1}{x+y}dydx$, where R is the region bounded by $x=0, y=0, x+y=1, x+y=4$, by using the mapping $T(u,v)=(u-uv,uv)$.

Problem 7 [10], Let D be the unit disk. Express $\int\int_D(1+x^2+y^2)^{\frac{3}{2}}dxdy$ as an integral over $[0,1]\times[0,2\pi]$ and evaluate.

Problem 8 [10], Use spherical coordinates to evaluate

$$\int_0^3\int_0^{\sqrt{9-x^2}}\int_0^{\sqrt{9-x^2-y^2}} \frac{(x^2+y^2+z^2)^{\frac{3}{2}}}{1+[x^2+y^2+z^2]^3}dzdydx.$$

Problem 9 [10], Let f be a function defined in $B=\{x^2+y^2+z^2<1\}$, suppose f is continuous at the origin and $f(0,0,0)=1$. Denote B_r be the ball of radius $r>0$ centered at the origin. Let $V(B_r)$ be the volume of B_r. Prove that

$$\lim_{r\to 0}\frac{1}{V(B_r)}\int\int\int_{B_r} f(x,y,z)dV = 1.$$

Problem 10 [10], Find improper integral $\int\int\int_{\mathbb{R}^3} f(x,y,z)dxdydz$, where

$$f(x,y,z) = \frac{1}{[1+(x^2+y^2+z^2)^{\frac{3}{2}}]^5}.$$