third Assignment, due on October 19, 2015.

Problem 1 [10], Consider the unit sphere S given by $x^2+y^2+z^2=1$. S intersects the z axis at two points. Which variables can we solve for near these points? What about the points of intersection of S with the y axis?

Problem 2 [10], Show that $x+zy+3yz^5=4$ is solvable for z as a function of (x,y) near $(0,1,1)$. Compute $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$ at $(0,1)$.

Problem 3 [10], Show that there are positive numbers $\delta>0$ and $\beta>0$ and unique function u and v from interval $(-1-\delta,-1+\delta)$ into the interval $(1-\beta,1+\beta)$ satisfying

$$xe^{u(x)}+u(x)e^{v(x)}=0=xe^{v(x)}+v(x)e^{u(x)}$$

for all x in the interval $(-1-\delta,-1+\delta)$ with $u(-1)=v(-1)=1$. (Hint: Implicit Function Theorem)

Problem 4 [10], Suppose $f,g,h:\mathbb{R}\to\mathbb{R}$ are differentiable. Show that the vector field $\mathbf{F}(x,y,z)=(f(x),g(y),h(z))$ is irrotational.

Problem 5 [10], Find the arc length of $\mathbf{c}(t)=\log t\mathbf{i}+t\mathbf{j}+2\sqrt{2t}\mathbf{k}$ for $1\le t\le 2$.

Problem 6 [10], Evaluate the integral $\int\int_R(ax^2+by+c)dxdy$ for $R=[0,1]\times[0,1]$.

Problem 7 [10], Let f be continuous on $R=[a,b]\times[c,d]$; for $a<x<b, c<y<b$, define

$$F(x,y)=\int_a^x\int_c^y f(u,v)dudv.$$

Show that $\frac{\partial^2 F}{\partial x\partial y}=\frac{\partial^2 F}{\partial y\partial x}=f(x,y)$. Use this example to discuss the relationship between Fubini's theorem and the equality of mixed partial derivatives.

Problem 8 [10], Reverse the iterated $\int_{-2}^2\int_0^{4-y^2}(4-x)dxdy$ as $\int_a^b\int_{f(x)}^{g(x)}(4-x)dydx$ for some constant a,b and some functions $f(x),g(x)$, then evaluate it.

Problem 9 [10], Evaluate $\int_0^1\int_0^{x^2}(x^2+xy-y^2)dydx$. Describe this iterated integral as an integral over a certain region D in the xy plane.

Problem 10 [10], Find $\int_0^4\int_{y/2}^2 e^{x^2}dxdy$.