First Assignment, due on October 5, 2015.

Problem 1[10], Compute an equation for the plane tangent to the graph of $f(x,y)=\frac{e^{x^2+y^2}}{\cos(x+y)}$ at $x=-2, y=2$.

Problem 2[10], Use chain rule to compute $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ if $f=\sin(u^2v), u=x+y$ and $v=x-y$.

Problem 3 [20], Let $f(x,y)=\frac{xy(x^2-y^2)}{x^2+y^2}$ if $(x,y)\neq(0,0)$ and $f(0,0)=0$.
(a), If $(x,y)\neq(0,0)$, calculate $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.
(b), show that $\frac{\partial f}{\partial x}(0,0)=\frac{\partial f}{\partial y}(0,0)=0$.
(c), show that $\frac{\partial^2 f}{\partial x\partial y}(0,0)=1$, $\frac{\partial^2 f}{\partial y\partial x}(0,0)=-1$.
(d), what went wrong? Why are the mixed partial derivatives are not equal?

Problem 4 [10], A function $f:\mathbb{R}\to\mathbb{R}$ is called an analytic function provided

$$f(x+h)=f(x)+f^{'}(x)h+\cdots+\frac{f^{(k)}(x)}{k!}h^k+\cdots=\sum_{k=0}^{\infty}\frac{f^{(k)}(x)}{k!}h^k$$

the series on the right-hand side converges and equals $f(x+h)$. Prove that, if f satisfies the following condition: On any closed interval $[a,b]$, there is a constant M (depending on $[a,b]$) such that for all $k=1,2,\cdots$, $|f^{(k)}(x)|\leq kM^k$ for all $x\in[a,b]$, then f is analytic.

Problem 5 [10], Let $f(x,y,z)=x^2+y^2+z^2+kyz$.
(a) Verify the $(0,0,0)$ is a critical point for f.
(b) Find all values of k such that f has a local minimum at $(0,0,0)$.

Problem 6 [10], Show that if $x_0=(x_1^0,x_2^0)$ is a critical point of a C^3 function f and

$$f_{11}(x_0)f_{22}(x_0)-f_{12}^2(x_0)<0,$$

then there are points x and $\tilde{x}$ near x_0 such that $f(x)>f(x_0)$ and $f(\tilde{x})<f(x_0)$.

Problem 7 [10], Find the absolute maximum and minimum value for the function $f(x,y)=e^{xy}$ on the rectangle R defined by $-1\leq x\leq 1, -1\leq y\leq 1$.

Problem 8 [10], Find the absolute maximum and minimum values for the function

$$f(x,y,z)=\sin(x^2+y^2+z^2+x+yz)$$

on the ball $B=\{(x,y,z)|x^2+y^2+z^2\leq 1\}$.

Problem 9 [10], Find the maximum and minimum of $f(x,y,z)=xy-y+x-1+z^2$ on the set $x^2+y^2+z^2\leq 3$.