MATH248 Midterm Test – (October 21, 2011)

Problem 1. Show that the set

$$D = \{(x, y)| - \infty < x < 0, 0 < y < 1\}$$

is open.

Problem 2. Show that the partial derivatives of the function

$$f(x,y) = \begin{cases} \frac{xy^2}{\sqrt{x^2+y^4}}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0), \end{cases} \tag{1}$$

exist everywhere. Is this function also differentiable everywhere?

Problem 3. Let B be the unit ball in $\mathbb{R}^3$ centered at the origin. Suppose $f : B \to \mathbb{R}$ is differentiable function. Prove that for any $X_1, X_2 \in B$, there is a point P on the line segment joining these two points such that

$$f(X_2) - f(X_1) =< \nabla f(P), X_2 - X_1 > .$$

Problem 4. Compute the divergence and curl of the vector field

$$\mathbf{F}(x, y, z) = x\mathbf{i} + (y + e^{xz})\mathbf{j} + (z + cos(x))\mathbf{k}.$$

Is $\mathbf{F}$ is a gradient vector field

Problem 5. Find the absolute minimum and maximum for the function $f(x, y, z) = z + x^2 + y^2 + z^2 - xy$ on the closed ball $\{(x, y, z)|x^2 + y^2 + z^2 \leq 1\}$.