How to Win Dots and Boxes

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Rules

Dots and Boxes

• Two players, Alice and Bob, start from a (rectangular) array of vertices (dots) and take turns to add edges horizontally or vertically.

• The player who completes the fourth side of a unit square (box) earns one point and takes another turn.

• The game ends when there is no more box can be completed. Whoever has more boxes is the winner (optimization).

• Note: there are two phases for this game (1. connecting vertices 2. collecting boxes).

Cr: https://en.wikipedia.org/wiki/Dots_and_Boxes
Let’s Play

Consider the following game board

- It’s Alice’s turn, she want to force Bob to play on the shorter chain.
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  • Choice 1: greedily take the chain, and open the long chain to Alice
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Suppose you are Bob, what is your next move?

• Choice 1: greedily take the chain, and open the long chain to Alice
• Choice 2: sacrifice 2 boxes, Alice is forced to open the long chain and then Bob can take the long chain
Double-Crossing

A Winning Strategy

• When Bob was forced to take a chain opened by Alice, he could close it with a double-cross move: sacrificing 2 boxes, but then Alice is forced to open a longer chain to Bob.

• After double-crossing, Bob gains control of the game. Otherwise, Alice has the control.

• The player who has control usually wins when there are several long chains.

• A long chain is a chain contains 3 or more boxes: it takes at least 3 boxes to complete a double-crossing move. (Note that a 2-box chain can be broken into 2 individual boxes, preventing the opponent from double-crossing.)
Long Chain
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In general, if there are more than 1 long chains, it is always a winning strategy to take the control by double-crossing.
• Let $m$ be the number of long chains and $n$ be the number of boxes, then if Bob use the strategy of double-crossing, he can score $n - 2m + 2 > 0$

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Definitions
Combinatorial Game Theory

- Combinatorial game theory studies two-player sequential games: players move sequentially as opposed to simultaneously in economic game theory.
- The winners in most combinatorial games depend on the last player, in contrast to Dots and Boxes.
- By convention \( L \) and \( R \) are used for each of the two players (instead of Alice and Bob).
- A game \( G \) is defined by \( G = \{ G^L | G^R \} \) where \( G^L \) and \( G^R \) stand for the set of left and right options respectively.
Game Tree

Combinatorial Game Theory

- For impartial games, the set of options for left and right are the same.
- We can draw a game tree of a position:
  - The root node is the original position
  - Create a node for each option from the root and connect to the root
  - For each node create node for its options and connect to the node
  - Repeat until there is not more options

Cr: https://en.wikipedia.org/wiki/Game_tree
More Definitions

Birthday of a Game

• Recall $G = \{ \mathcal{G}^L | \mathcal{G}^R \}$ where $\mathcal{G}^L$ and $\mathcal{G}^R$ stand for the set of left and right options.

• The *birthday* of a game $G = \{ \mathcal{G}^L | \mathcal{G}^R \}$ is defined as $1 + \max$ birthday of any game in $\mathcal{G}^L \cup \mathcal{G}^R$.

• Base case: if $\mathcal{G}^L = \mathcal{G}^R = \emptyset$, then the birthday of $G$ is 0, i.e. $0 = \{ \mid \}$.

• Apply the definition recursively we have:
  • $1 = \{0 \mid \}$
  • $-1 = \{ \mid 0 \}$
  • $* = \{0 \mid 0 \}$
Thanks