## ASSIGNMENT 9 - NUMBER THEORY, WINTER 2009 (LAST ASSIGNMENT!)

## Submit by Monday, March 30, 16:00.

Solve following questions:

- (40) Factor n = 3599, using both Fermat's and Pollard's p = 1 methods. In Pollard's method you can calculate the gcd's using a computer.
- (41) The message 2401 was sent using RSA with modulus n = 3599 and encryption key e = 2983. What was the original message?

In the following questions use the method we used in class for calculating  $N(x^2 + y^2 = 1)$ . Namely,  $N(x^2 + y^2 = 1) = \sum_{a+b=1} \left(1 + \left(\frac{a}{p}\right)\right) \left(1 + \left(\frac{b}{p}\right)\right)$ , etc.

- (42) Find the number of solutions to the equation  $x^2 + y^2 = 0$  over  $\mathbb{F}_p$ .
- (43) Find the number of solutions to the equation  $x^2 + y^2 = \alpha$  over  $\mathbb{F}_p$ , where  $\alpha \in \mathbb{F}_p$  is a non-zero constant.

The honors students need to submit also the following problem.

(M) Find the number of solutions to the equation  $Ax^2 + By^2 = \alpha$  over  $\mathbb{F}_p$ , where  $\alpha \in \mathbb{F}_p$  is a constant and  $A, B \in \mathbb{F}_p$  are non-zero constants.