ASSIGNMENT 6 - NUMBER THEORY, WINTER 2009

Submit by Monday, March 2, 16:00.

Solve following questions:

(25) Prove that the following list of primes (in which the next prime is smaller than twice its predecessor) shows that Bertrand's Postulate is true for $n < 2^{10}$.

$$2, 3, 5, 7, 13, 23, 43, 83, 163, 317, 631, 1259.$$

(26) By studying the proof of Bertrand's Postulate, find an explicit function $f(n)$ such that $f(n) \to \infty$ as $n \to \infty$ and $f(n) \leq \sharp \{p : p \text{ is prime }, n < p \leq 2n\}$.

(27) In the course of proving Tchebychev's theorem we found that $\pi(x) \geq \log(2) \cdot \frac{x}{\log(x)} + O(1)$. Estimate this $O(1)$ and find a constant A (as close to $\log(2)$ as you can) such that $A \cdot \frac{x}{\log(x)} \leq \pi(x)$ for all $x \geq 100$, say.

(28) Prove that the Prime Number Theorem is equivalent to

$$\pi(x) \sim \mathrm{Li}(x).$$

(This is really an easy exercise in the definitions of assymptotic and big O).

The honors students do not need to submit additional problems this time.