# COURSE SYLLABUS FOR MATH 570/571, 2011

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*This is an "ideal" syllabus. We will have to see if we can actually cover and what will be deferred to the next term.*

(1) Groups – Selected topics.
   (a) Group actions on sets. (Definition, reformulation as $G \to \Sigma_S$, orbits, stabilizers, Cauchy-Frobenius Formula, revisiting Lagrange's theorem, the coset representation....)
   (b) The class equation, Sylow's theorems, and some basic results on $p$-groups. Applications (groups of order $pq, pq^2$, solvability of groups of order less than 60).
   (c) Simplicity of $A_n$ and $\mathrm{PSL}_n(\mathbb{F}_q)$.
   (d) Solvable groups.
   (e) Free groups and free products of groups and the concepts of category, functor, adjoint functors. Universal property in terms of initial/final object.
(2) Modules, Part I.
   (a) A recall of the basic definitions and theorems for modules.
   (b) Modules over PID and applications.
   (c) Localization.
   (d) Free modules.
(3) Categories.
   (a) Recall: The language of categories, functors and universal objects.
   (b) Equivalence of categories.
   (c) Injective and projective limits.
(4) Fields.
   (a) Recall: The fundamental theorem of Galois theory and some basic results in Galois theory.
   (b) Profinite groups and, in particular, $\mathbb{Z}_p$.
   (c) Infinite Galois theory.
   (d) Finite fields and cyclotomic fields.
   (e) Kummer theory.
   (f) Solvability by radicals.
   (g) Calculation of Galois groups.
(5) Rings, Part I.
   (a) The spectrum of a ring.
   (b) Integral extensions and the going-up and going-down theorems.
   (c) Noether's normalization lemma, and Hilbert's Nullstellensatz.
   (d) Noetherian and Artinian rings.
   (e) Hilbert's basis theorem.
   (f) Dedekind rings.
(6) Modules, Part II.
   (a) Tensor products.

(b) Exact and adjoint functors.
(c) Projective modules.
(d) Injective modules.
(e) Flat modules.

(7) Rings, Part II.
(a) The Jacobon radical.
(b) Nakayam's lemma.
(c) Semisimple rings and modules.
(d) Jacobson's density theorem and the Artin-Wedderburn theorem.

(8) Linear representations of finite groups.
(a) Linear representations of groups.
(b) Maschke's theorem.
(c) Characters. Orthogonality of characters. Frobenius Reciprocity.
(d) Representations of nilpotent groups.
(e) Representations of the symmetric group.
(f) Representations of $\mathrm{GL}_2(\mathbb{F})$, for $\mathbb{F}$ a finite field.