COURSE SYLLABUS FOR MATH 570, 2010

EYAL GOREN

This is an "ideal" syllabus. We will have to see if we can actually cover all that in a year. It depends, to an extent, on the students' previous preparation and ability to work independently.

(1) Categories, Part I.
   (a) The language of categories.
   (b) Universal objects.
(2) Groups, Part I.
   (a) A recall at light speed of groups, up to the Syllow theorem.
   (b) The class equation and some basic results on p-groups.
   (c) The Syllow theorems.
   (d) Nilpotent and Solvable groups.
   (e) Presentations of groups. The Stallings graph and applications.
   (f) Free and amalgamated products of groups.
(3) Modules, Part I.
   (a) A recall at light speed of the basic definitions and theorems for modules.
   (b) Localization.
   (c) Free modules.
   (d) Modules over PID and applications.
(4) Categories, Part II.
   (a) Equivalence of categories.
   (b) Injective and projective limits.
(5) Fields.
   (a) Recall of some basic facts and results about field extensions.
   (b) Separable and normal extensions.
   (c) Galois extensions.
   (d) The fundamental theorem of Galois theory.
   (e) Finite fields and cyclotomic fields.
   (f) Calculation of Galois groups.
   (g) A little about infinite Galois theory.
(6) Rings, Part I.
   (a) Integral extensions and the going-up and going-down theorems.
   (b) Noether's normalization lemma, and Hilbert's Nullstellensatz.
   (c) Noetherian and Artinian rings.
   (d) Hilbert's basis theorem.
(7) Categories, Part III.
   (a) Exact functors.
   (b) Adjoint functors.
(8) Modules, Part II.
   (a) Tensor products.

(b) Projective modules.
(c) Injective modules.
(d) Flat modules.

(9) Rings, Part II.
(a) The Jacobon radical.
(b) Nakayam's lemma.
(c) Semisimple rings and modules.
(d) Jacobson's density theorem and the Artin-Wedderburn theorem.

(10) Groups, Part II.
(a) Linear representations of groups.
(b) Maschke's theorem.
(c) Characters. Orthogonality of characters. Frobenius Reciprocity.
(d) Representations of nilpotent groups.
(e) Representations of the symmetric group.
(f) Representations of $\mathrm{GL}_2(\mathbb{F})$, for $\mathbb{F}$ a finite field.

(11) Homological algebra.
(a) The snake lemma and the 5 term lemma.
(b) Projective and injective resolutions and derived functors.
(c) The derived functors Tor and Ext.
(d) Homotopy and independence on the resolution.