MATH 381 FALL 2012 PRACTICE FINAL

1. Compute
$$\int \frac{\cosh e^z}{z^2 - 4z + 3}\,\mathrm{d}z,$$
around the (positively oriented) square with corners at $z = 3 \pm 1$, and $z = 3 \pm i$.
2. Find the radius of convergence of the Taylor series of
$$f(z) = \frac{1}{z^{1/2} - 1},$$
expanded about $z = 2$, where we take the principal branch of the square root function.
3. Determine the interval along the x-axis for which the real Taylor series of
$$f(x) = \frac{1}{x^4 + 16},$$
centred about $x = 2$, converges to $f(x)$.
4. Find the Taylor series for $f(z) = \dfrac{1}{z^3}$ expanded about $z = 1$.
5. Find the Taylor series for
$$f(z) = \frac{e^z}{(z-2)(z+1)},$$
expanded about $z = 0$.
6. Obtain the Laurent series for
$$f(z) = \operatorname{Log}\left(1 + \frac{2}{z-2}\right),$$
expanded in powers of $z - 2$.
7. For each of the functions
$$f(z) = \frac{\sin z}{z^{10}(z+1)^2}, \qquad \text{and} \qquad f(z) = \frac{\cos(1/z)}{\sin z},$$
state the location and order of each pole and find the corresponding residue.
8. Using residues, evaluate
$$\int \frac{\mathrm{d}z}{\sin(z^{1/2})},$$
along the circle $|z - 9| = 5$, positively oriented. Take the principal branch of the square root function.
9. Evaluate
$$\int_0^\infty \frac{x^4}{x^6 + 1}\,\mathrm{d}x, \qquad \text{and} \qquad \int_{-\infty}^\infty \frac{(x-1)\cos(2x)}{x^2 + x + 1}\,\mathrm{d}x.$$
10. Find the Cauchy principal value of each of
$$\int_{-\infty}^\infty \frac{\cos 2x}{x^2 - 16}\,\mathrm{d}x, \qquad \text{and} \qquad \int_{-\infty}^\infty \frac{\cos x}{(x - \frac{\pi}{2})(x^2 + 1)}\,\mathrm{d}x.$$