1. Solve \((3y \cos x + xe^x + e^x)dx + (3\sin x + 2y)dy = 0\).

**Solution:** The equation is of the form

\[ a(x, y)dx + b(x, y)dy = 0, \]

where

\[ a(x, y) = 3y \cos x + xe^x + e^x, \quad \text{and} \quad b(x, y) = 3\sin x + 2y. \]

We have

\[ \frac{\partial a}{\partial y} = 3 \cos x + 0, \quad \text{and} \quad \frac{\partial b}{\partial x} = 3\cos x + 0, \]

so the equation is *exact*. This means that there is a function \(F(x, y)\) such that

\[ \frac{\partial F(x, y)}{\partial x} = a(x, y), \quad \text{and} \quad \frac{\partial F(x, y)}{\partial y} = b(x, y). \]

The latter condition gives

\[ F(x, y) = \int b(x, y)dy = \int (3\sin x + 2y)dy = 3y \sin x + y^2 + f(x), \]

and substituting this into the former, we get

\[ \frac{\partial F(x, y)}{\partial x} = 3y \cos x + f'(x) = a(x, y) = 3y \cos x + xe^x + e^x. \]

Now we need to find a function \(f(x)\) such that \(f'(x) = xe^x + e^x\). This can be done either by the direct observation \((xe^x)' = e^x + xe^x\), or by integration

\[ f(x) = \int (xe^x + e^x) \, dx = xe^x + C, \]

which can be computed with the help of integration by parts

\[ \int xe^x \, dx = \int x \, de^x = xe^x - \int e^x \, dx = xe^x - e^x + C. \]

Since we just need one function \(f(x)\) such that \(f'(x) = xe^x + e^x\), we pick the simplest choice \(f(x) = xe^x\), which yields

\[ F(x, y) = 3y \sin x + y^2 + xe^x. \]

Finally, we can write the solution in implicit form as

\[ 3y \sin x + y^2 + xe^x = A, \]

with \(A\) an arbitrary constant.
2. Solve the initial value problem \( xy' = xe^{-y/x} + y \) with \( y(1) = 0 \).

**Solution:** Let us write the equation as

\[
y' = e^{-y/x} + y/x.
\]

We immediately recognize this as a *homogeneous equation*, because the right hand side \( f(x, y) = e^{-y/x} + y/x \) satisfies

\[
f(\lambda x, \lambda y) = e^{-(\lambda y)/(\lambda x)} + \frac{\lambda y}{\lambda x} = e^{-y/x} + \frac{y}{x} = f(x, y), \quad \text{for any } \lambda \neq 0.
\]

We are going to carry out the substitution \( y = ux \), where \( u = u(x) \) is the new dependent unknown that replaces \( y(x) \). Note that in terms of \( u \), the initial condition \( y(1) = 0 \) becomes \( u(1) = y(1)/1 = 0 \). Taking into account \( y' = u'x + u \), we get

\[
u'x + u = e^{-u} + u.
\]

After some straightforward manipulations, we end with the separable equation

\[
e^u u' = \frac{1}{x},
\]

which yields

\[
e^{u(x)} = \log |x| + C.
\]

At this point, it is convenient to use the initial condition \( u(1) = 0 \), that is,

\[
\overset{0}{u} = \overset{1}{\log 1 + C},
\]

to settle the value \( C = 1 \) for \( C \). Therefore, we can write

\[
u(x) = \log(\log |x| + 1),
\]

and so the final solution is

\[
y(x) = xu(x) = x \log(\log |x| + 1).
\]