MATH 315 WRITTEN ASSIGNMENT 2

DUE WEDNESDAY APRIL 2

1. Find all solutions of the equation
$$y\cos y\,\mathrm{d}x + x(\cos y + y\sin y - y\cos y)\,\mathrm{d}y = 0.$$
2. Show that every solution of the constant coefficient equation
$$y'' + py' + qy = 0,$$
tends to 0 as $x \to \infty$ if, and only if, the real parts of the roots of the characteristic equation
$$r^2 + pr + q = 0,$$
are negative.
3. Suppose that y_1 and y_2 are linearly independent solutions of the constant coefficient equation
$$y'' + py' + qy = 0,$$
and let $W(x) = y_1(x)y_2'(x) - y_1'(x)y_2(x)$ be the Wronskian of y_1 and y_2. Show that W is constant if and only if $p = 0$.
4. Solve the initial value problem
$$y'' - 6y' + 8y = 3e^x + 2x^2, \qquad y(0) = 1, \qquad y'(0) = 0.$$
5. Find all solutions of the equation
$$x^2y'' - 5xy' + 9y = x^3, \qquad \text{for} \quad x > 0.$$
6. Find all solutions of the equation
$$y'' - 2y' + y = \frac{e^x}{4 + x^2}.$$
7. Compute the Laplace transform of
$$f(t) = \begin{cases} 1, & 0 \le t < 1, \\ 0, & 1 \le t < \pi, \\ \sin 2t, & t \ge \pi. \end{cases} \qquad (*)$$
8. By using the Laplace transform method, solve the initial value problem
$$y'' + 4y = f(t), \qquad y(0) = 0, \qquad y'(0) = 1,$$
where $f(t)$ is given by $(*)$. Sketch the graphs of $f(t)$ and the solution $y(t)$.

Date: April 1, 2014.