Problems from the Midterm Exam

Math 222 Winter 2015

1. Derive the Maclaurin series for the following functions. (cf. Practice Problem 4)
   (a) $L(x) = \int_0^x \frac{\log(1+t)}{t}\,\mathrm{d}t.$
   (b) $C(x) = \int_0^x \frac{1-\cos t}{t^2}\,\mathrm{d}t.$
   (c) $S(x) = \int_0^x \sin t^2\,\mathrm{d}t.$
   (d) $S(x) = \int_0^x \frac{e^t - e^{-t}}{t}\,\mathrm{d}t.$
   (e) $C(x) = \int_0^x \frac{e^t + e^{-t} - 2}{t^2}\,\mathrm{d}t.$

2. Decide if the following series converge. (cf. Practice Problem 5)
   (a) $\sum_{n=1}^{\infty} \log(n \sin \frac{1}{n}).$
   (b) $\sum_{n=1}^{\infty} \log \frac{e^{\frac{1}{n}} + e^{-\frac{1}{n}}}{2}.$
   (c) $\sum_{n=1}^{\infty} \log(n \arctan \frac{1}{n}).$
   (d) $\sum_{n=2}^{\infty} \log \frac{n^2}{n^2-1}.$

3. Determine the convergence radii of the following power series. (cf. Practice Problem 7)
   (a) $\sum_{n=0}^{\infty} (\log n + a^n) x^n.$
   (b) $\sum_{n=0}^{\infty} (\sqrt{n} + a^n) x^n.$

4. Compute the unit tangent $T(t)$, the principal unit normal $N(t)$, and the curvature $\kappa(t)$, for each of the following plane curves.
   (a) $X(t) = (\frac{\cos t}{t}, \frac{\sin t}{t}), \qquad t > 0.$
   (b) $X(t) = (t^2 \cos t, t^2 \sin t), \qquad t \geq 0.$
   (c) $X(t) = (e^t \cos t, e^t \sin t), \qquad -\infty < t < \infty.$
   (d) $X(t) = (t - \sin t, 1 - \cos t), \qquad 0 < t < 2\pi.$
   (e) $X(t) = (\cos^3 t, \sin^3 t), \qquad 0 < t < \frac{\pi}{2}.$