ERRATA FOR PROBLEMS AND SOLUTIONS SUBMITTED BY STUDENTS

MATH 222 WINTER 2015

Problem 1 (page 1)

There are some errors in the solution of Problem 1. We will give here a full solution.

Problem statement: Derive the Maclaurin series for $C(x) = \int_0^x \cos^2 t \, \mathrm{d}t$.

Solution: The idea is to use the identity

$$\cos^2 t = \frac{1 + \cos 2t}{2}, \tag{1}$$

which can be deduced from the double angle formula

$$\cos 2t = \cos^2 t - \sin^2 t = \cos^2 t - (1 - \cos^2 t) = 2\cos^2 t - 1. \tag{2}$$

By substituting $x = 2t$ into the Maclaurin series

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}, \tag{3}$$

we get

$$\cos 2t = \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n}}{(2n)!} t^{2n}, \tag{4}$$

and hence

$$\begin{aligned} \cos^2 t &= \frac{1}{2}\Big(1 + \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n}}{(2n)!} t^{2n}\Big) = \frac{1}{2}\Big(1 + 1 + \sum_{n=1}^{\infty} \frac{(-1)^n 2^{2n}}{(2n)!} t^{2n}\Big) \\ &= 1 + \sum_{n=1}^{\infty} \frac{(-1)^n 2^{2n-1}}{(2n)!} t^{2n}. \end{aligned} \tag{5}$$

Finally, by termwise integration, we find

$$\begin{aligned} C(x) = \int_0^x \cos^2 t \, \mathrm{d}t &= x + \sum_{n=1}^{\infty} \frac{(-1)^n 2^{2n-1}}{(2n+1)!} x^{2n+1} \\ &= x - \frac{2}{3!}x^3 + \frac{2^3}{5!}x^5 - \frac{2^5}{7!}x^7 + \dots. \end{aligned} \tag{6}$$

Problem 6 (page 6)

The final reparameterization should be

$$t = \frac{1}{2}\ln\Big(\frac{s + \sqrt{2}}{\sqrt{2}}\Big), \qquad \text{or} \qquad t = \frac{1}{2}\ln\Big(\frac{s}{\sqrt{2}} + 1\Big), \tag{7}$$

but *not* $t = \frac{1}{2}\ln\big(\frac{s+1}{\sqrt{2}}\big)$.

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Problem 8 (page 8)

The third line of the formula

$$\begin{aligned}\sin^2(x) &= \frac{1}{2}\Big(1 - \sum_{n=0}^{\infty}\frac{(-1)^n(2x)^{2n}}{(2n)!}\Big)\\ &= \frac{1}{2}\Big(1 - \Big(1 + \sum_{n=1}^{\infty}\frac{(-1)^n(2^{2n})(x^{2n})}{(2n)!}\Big)\Big)\\ &= \frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^n(2^{2n})(x^{2n})}{(2n)!},\end{aligned} \tag{8}$$

is missing a minus sign, so the final answer should be

$$\sin^2(x) = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}(2^{2n-1})}{(2n)!}x^{2n}. \tag{9}$$

It makes sense because the first term of the expansion of $\sin^2(x)$ comes from squaring the first term of $\sin x = x - \frac{1}{6}x^3 + \dots$, hence should be x^2.

Problem 9 (page 9)

The last displayed formula in the solution should be

$$X(t(10)) = X(1.33) = \langle 4(1.33)^3, 2 - (1.33)^2, (1.33)^3\rangle. \tag{10}$$

In the posted solution, $t = 1.33$ was erroneously plugged into $X'(t)$ instead of $X(t)$.

Problem 12 (page 12)

There are some sign issues for the list of derivatives at $x = 1$. They should be

$$\begin{aligned} &f^{(0)}(1) = 1,\\ &f^{(1)}(1) = -2,\\ &f^{(2)}(1) = 6,\\ &f^{(3)}(1) = -24,\\ &\quad\dots\\ &f^{(n)}(1) = (-1)^n(n+1)!.\end{aligned} \tag{11}$$

Then for the final answer, we should have

$$\frac{1}{x^2} = \sum_{n=0}^{\infty}\frac{f^{(n)}(1)}{n!}(x-1)^n = \sum_{n=0}^{\infty}\frac{(-1)^n(n+1)!}{n!}(x-1)^n = \sum_{n=0}^{\infty}(-1)^n(n+1)(x-1)^n. \tag{12}$$

Problem 13 (page 13)

For $T(\frac{\pi}{4})$, the vector is written as $\langle 0, 3, \frac{4}{5}\rangle$, but it should be $\langle 0, \frac{3}{5}, \frac{4}{5}\rangle$. There is also a mistake in the computation of the cross-product $B = T \times N$, where the negative sign for the second component of B is not taken into account. The correct expression is

$$B(t) = \langle \tfrac{4}{5}\sin(-3t), -\tfrac{4}{5}\cos(-3t), -\tfrac{3}{5}\rangle, \qquad \text{and hence} \qquad B(\tfrac{\pi}{3}) = \langle 0, +\tfrac{4}{5}, -\tfrac{3}{5}\rangle. \tag{13}$$

Problem 14 (page 14)

Part (a), the last line should be

$$\frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}. \tag{14}$$

Problem 18 (page 22)

In equation (86), the third coordinate of $X'(t)$ should be $-\frac{t^3}{24}$, *not* $-\frac{t^3}{32}$, because

$$\frac{\mathrm{d}}{\mathrm{d}t}\Big(-\frac{t^4}{96}\Big) = -\frac{4t^3}{96} = -\frac{t^3}{24}. \tag{15}$$

The rest of the solution should be corrected accordingly.

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