AIPW WITH A CONTINUOUS EXPOSURE

The inverse probability weighting (IPW) estimation for a continuous treatment uses IPW estimators with weights that target the outcome at (potential) value z of the form

$$w(x,t) = \frac{\mathbb{1}_{(z-d,z+d)}(t)}{f_{Z|X}(t|x)}$$

or via the stabilized version, that is

$$w(x,t) = \frac{\mathbb{1}_{(z-d,z+d)}(t) f_Z(t)}{f_{Z|X}(t|x)}.$$

to yield the $\widehat{\mu}_{\text{IPW}}(z)$ estimator

$$\widehat{\mu}_{\text{IPW}}(z) = \sum_{i=1}^{n} W_{iz} Y_i$$

with

$$W_{iz} = \frac{w(x_i, z_i)}{\sum\limits_{j=1}^{n} w(x_j, z_j)}.$$

To extend this to the augmented (AIPW) estimator, we propose and fit a mean model $m(x,z)$ to form the estimator

$$\widehat{\mu}_{\text{AIPW}}(z) = \sum_{i=1}^{n} W_{iz}(Y_i - m(X_i, Z_i)) + \frac{1}{n}\sum_{i=1}^{n} m(X_i, z)$$

EXAMPLE: In this example, we have a data generating process with $k = 10$ predictors, joint Normally distributed $X \sim Normal(\boldsymbol{\mu}, \Sigma)$, with treatment model given by

$$Z|X = x \sim Normal(\mathbf{x}_\alpha \alpha, \sigma_Z^2)$$

and outcome model

$$Y|X = x, Z = z \sim Normal(\mu(x,z), \sigma_Y^2)$$

with

$$\mu(x,z) = \mathbf{x}_0 \beta + \psi_0 z = \beta_0 + \sum_{l=1}^{k} x_l \beta_l + \psi_0 z.$$

In the model, all predictors are predictors of Z, but only first three components are confounders. In the analysis

$$\alpha = (4, 1, 1, 1, 1, 1, 1, 1, -2, -2, 2)^\top \qquad \beta = (2, -2, 2.2, 3.6, 0, 0, 0, 0, 0, 0)^\top$$

and $\psi_0 = 1$. Here, $\sigma_Z = 5$ and $\sigma_Y = 1$. The quantity ψ_0 measures the unconfounded effect of treatment, as, marginally,

$$\mathbb{E}[Y(z)] = \psi_0 z + \mu_0 \beta.$$

where $\mu_0 = (1, \boldsymbol{\mu}^\top)$. In the studies below, we consider

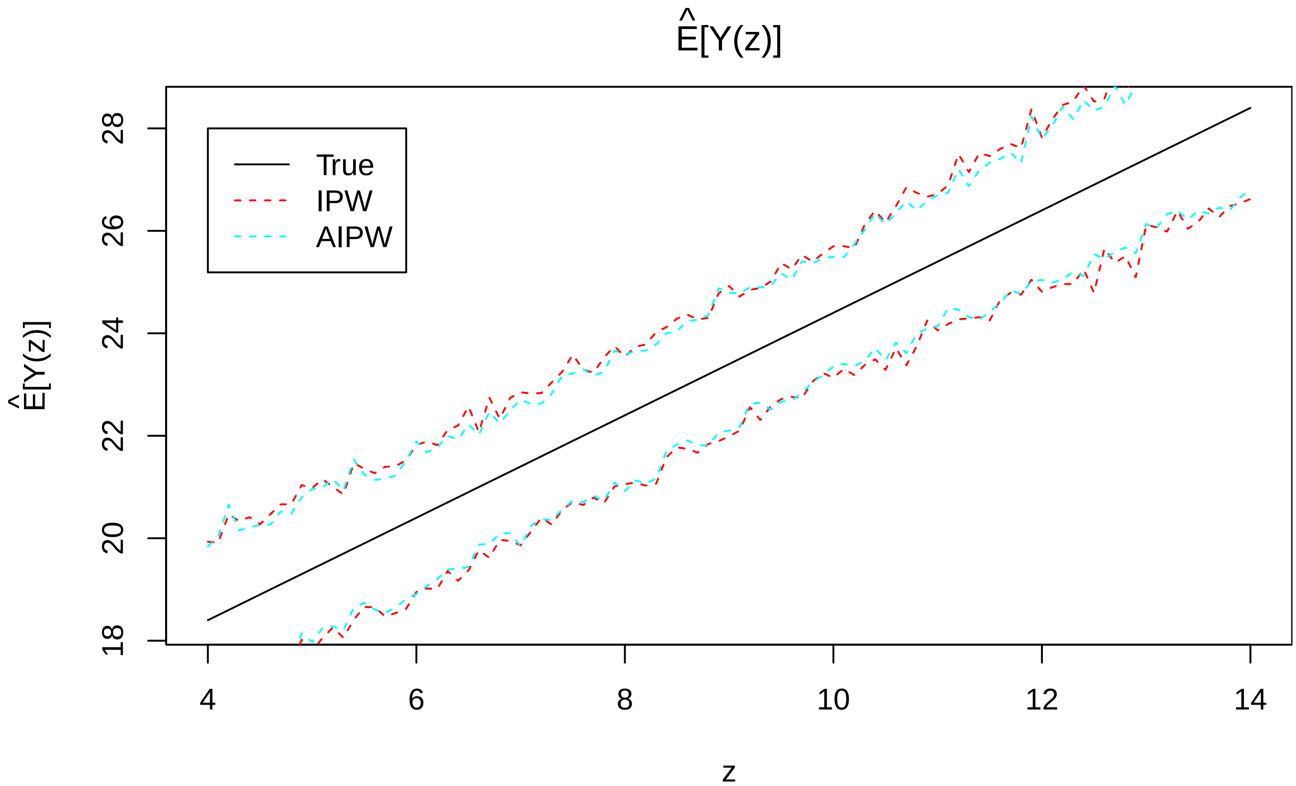
- IPW with ordinary weights
- IPW with stabilized weights
- AIPW with ordinary weights
- AIPW with stabilized weights

where for the AIPW methods, the (mis-specified) mean model

$$m(x,z) = \beta_0 + \beta_1 x_1 + \psi_0 z$$

is used. In each case, the weights are standardized to sum to 1 as in the formula for W_{iz}.

We carry out a simulation study of 200 replicates with $n = 1000$.

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+               Estimate  Std. Error  t value      Pr(>|t|)
+ (Intercept) 14.354911 0.021889016 655.8043 6.628522e-182
+ zgrid        1.005962 0.002313741 434.7773 3.071856e-164
+               Estimate  Std. Error  t value      Pr(>|t|)
+ (Intercept) 14.382369 0.017013575 845.3467 8.082911e-193
+ zgrid        1.002121 0.001798391 557.2323 6.646506e-175
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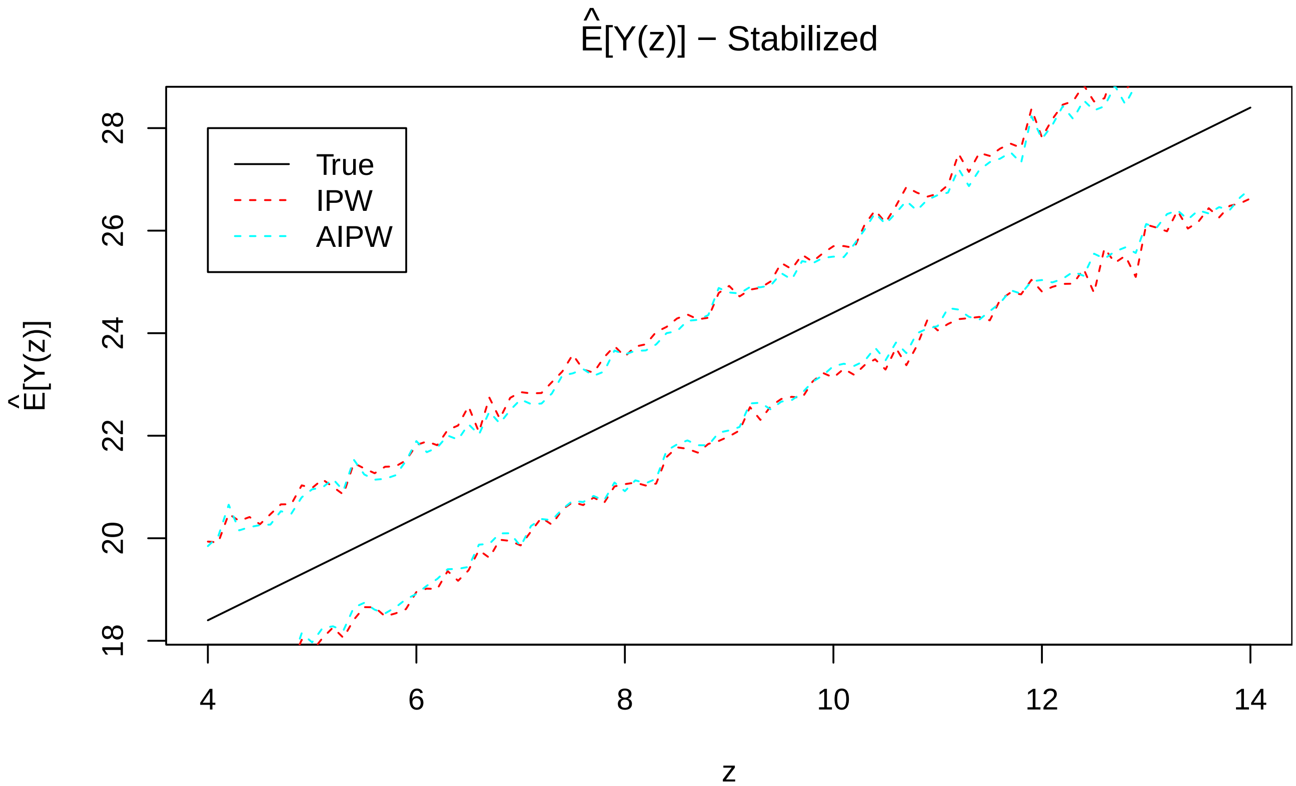


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+                Estimate  Std. Error  t value      Pr(>|t|)
+ (Intercept) 14.355425 0.021926544 654.7053 7.825479e-182
+ zgrid        1.005915 0.002317708 434.0129 3.656127e-164
+                Estimate  Std. Error  t value      Pr(>|t|)
+ (Intercept) 14.383584 0.017038696 844.1716 9.276154e-193
+ zgrid        1.001964 0.001801046 556.3233 7.811980e-175
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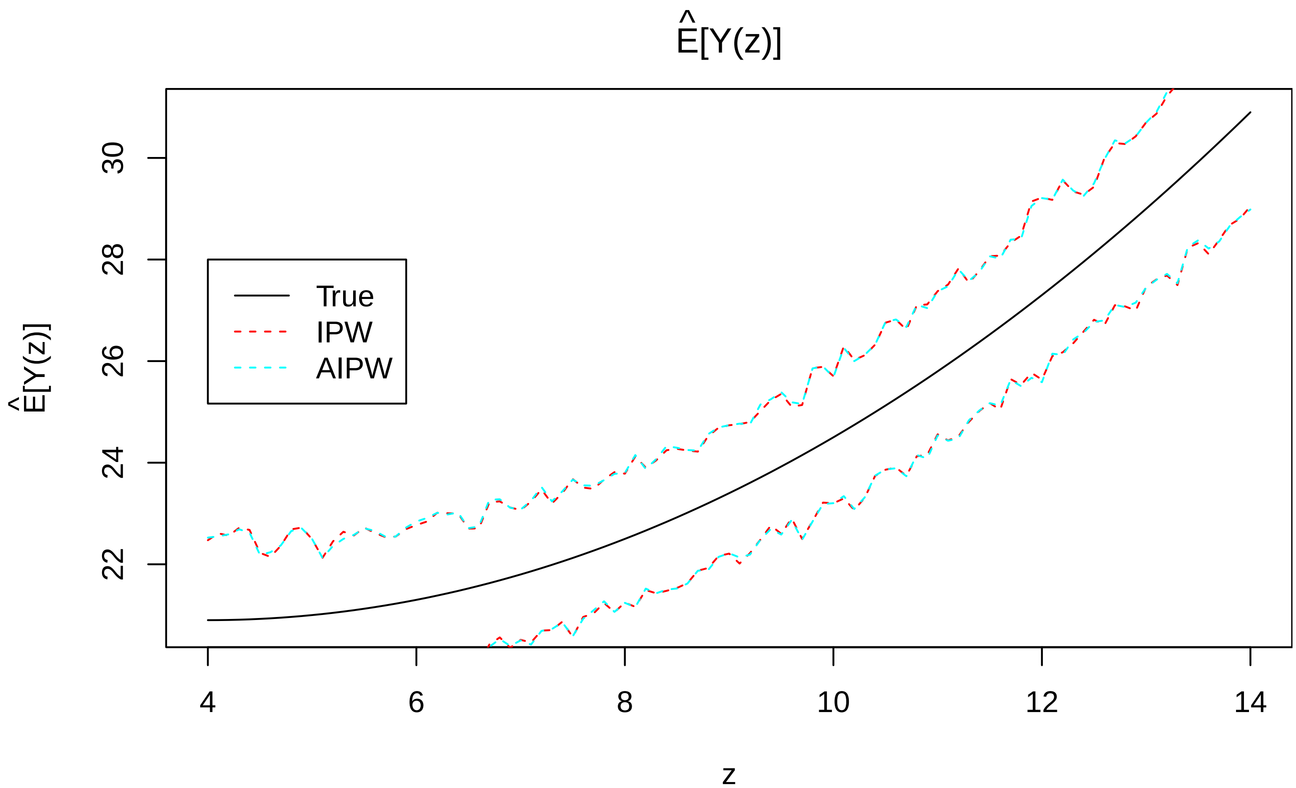
EXAMPLE: The methods also work for quadratic data generating models

$$\mu(x, z) = \mathbf{x}_0\beta + \psi_0 z + \psi_1 z^2$$

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+                Estimate   Std. Error   t value      Pr(>|t|)
+ (Intercept) 22.4756008 0.0597036807 376.45252 1.099343e-156
+ zgrid       -0.8032958 0.0141794935 -56.65194  1.063100e-76
+ I(zgrid^2)   0.1007860 0.0007796134 129.27690 2.648914e-111
+                Estimate   Std. Error   t value      Pr(>|t|)
+ (Intercept) 22.4607210 0.0625164183 359.27716 1.064649e-154
+ zgrid       -0.7995647 0.0148475125 -53.85176  1.310781e-74
+ I(zgrid^2)   0.1006004 0.0008163422 123.23314 2.808559e-109
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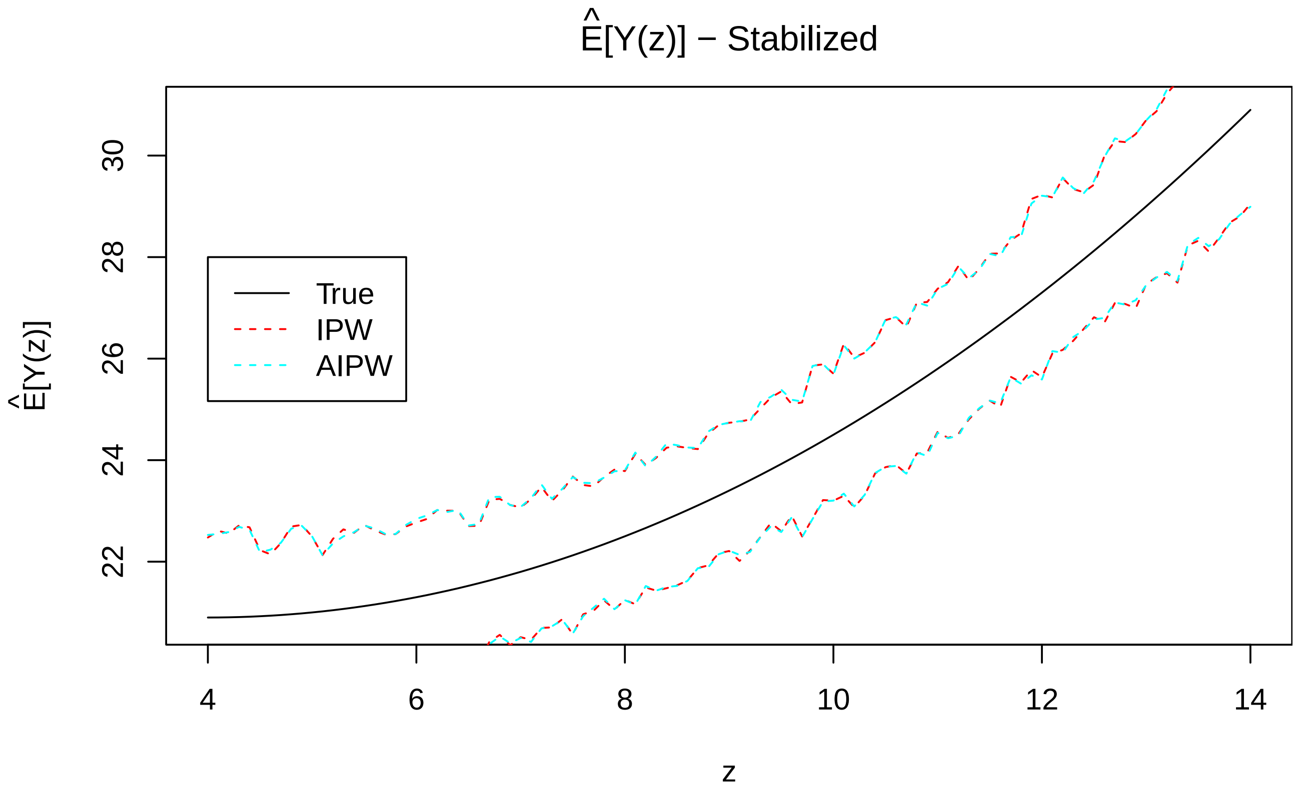


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+                Estimate  Std. Error   t value      Pr(>|t|)
+ (Intercept) 22.4741897 0.059523687 377.56716 8.229730e-157
+ zgrid       -0.8029058 0.014136745 -56.79566  8.353975e-77
+ I(zgrid^2)   0.1007613 0.000777263 129.63599 2.021578e-111
+                Estimate   Std. Error   t value      Pr(>|t|)
+ (Intercept) 22.4603427 0.0622188531 360.98934 6.683598e-155
+ zgrid       -0.7993801 0.0147768414 -54.09682  8.521555e-75
+ I(zgrid^2)   0.1005844 0.0008124566 123.80277 1.792450e-109
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