PROPENSITY SCORE REGRESSION WITH CONTINUOUS EXPOSURE

EXAMPLE: In this example, we have a data generating process with $k = 10$ predictors, joint Normally distributed $X \sim Normal(\boldsymbol{\mu}, \Sigma)$, with treatment model given by

$$Z|X = x \sim Normal(\mathbf{x}_\alpha \alpha, \sigma_Z^2)$$

and outcome model

$$Y|X = x, Z = z \sim Normal(\mu(x,z), \sigma_Y^2)$$

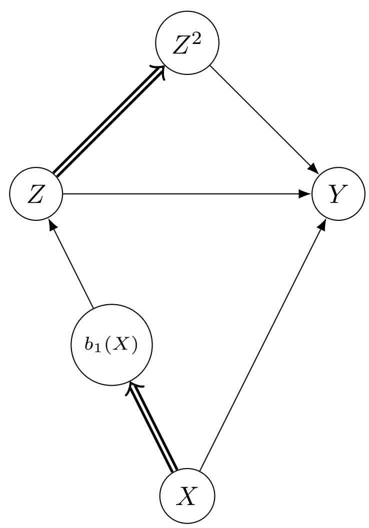
with

$$\mu(x,z) = \mathbf{x}_0\beta + \psi_0 z + \psi_1 z^2 = \beta_0 + \sum_{l=1}^{k} \beta_l x_l + \psi_0 z + \psi_1 z^2.$$

In the model, all predictors are predictors of Z, but only first three components are confounders. In the analysis

$$\alpha = (4,1,1,1,1,1,1,1,-2,-2,2)^\top \qquad \beta = (2,-2,2.2,3.6,0,0,0,0,0,0,0)^\top$$

with treatment effect parameters $\psi_0 = 1$, $\psi_1 = 1$. Here, $\sigma_Z = 2$ and $\sigma_Y = 10$.



The quantities ψ_0 and ψ_1 measure the unconfounded effect of treatment, as, marginally,

$$\mathbb{E}[Y(z)] = \psi_0 z + \psi_1 z^2 + \mu_0\beta.$$

We have that the ATE curve is

$$\mathbb{E}[Y(z) - Y(0)] = \psi_0 z + \psi_1 z^2.$$

Propensity score regression can be carried out using balancing scores that block the backdoor paths. For up to quadratic terms, we may need to model

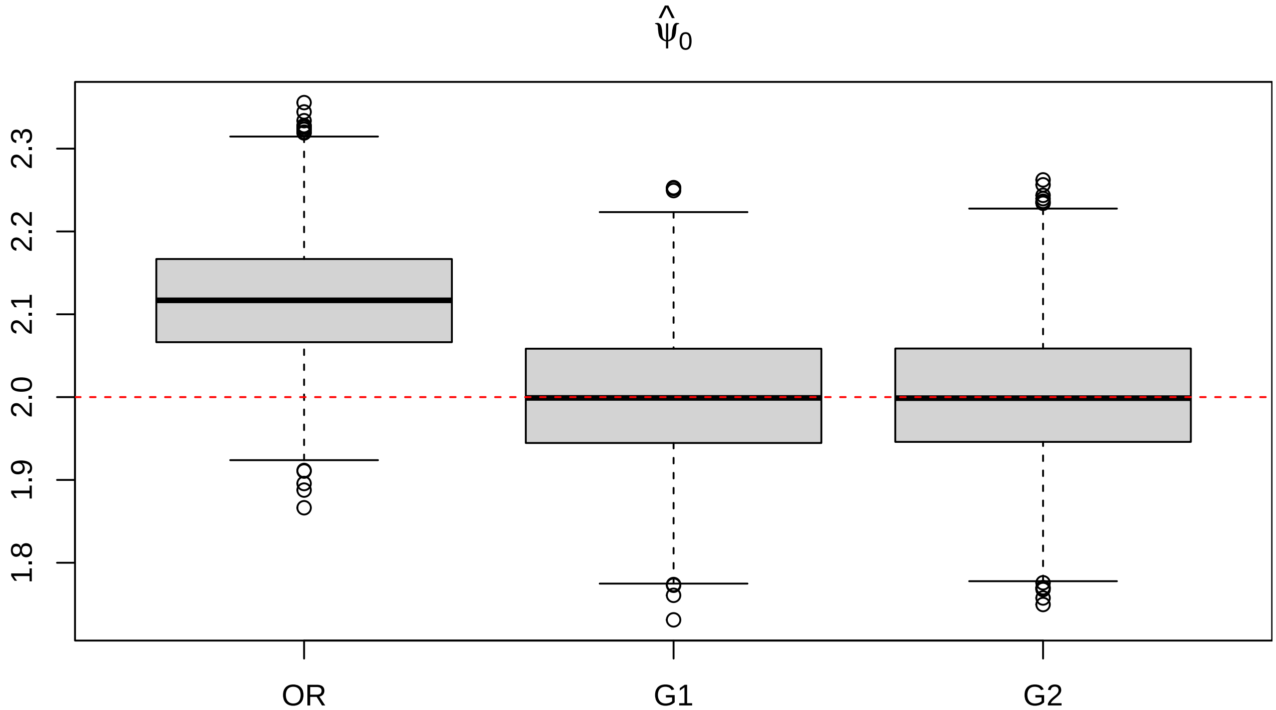
$$b_1(x) = \mathbb{E}_{Z|X}[Z|X = x] \qquad b_2(x) = \mathbb{E}_{Z|X}[Z^2|X = x]$$

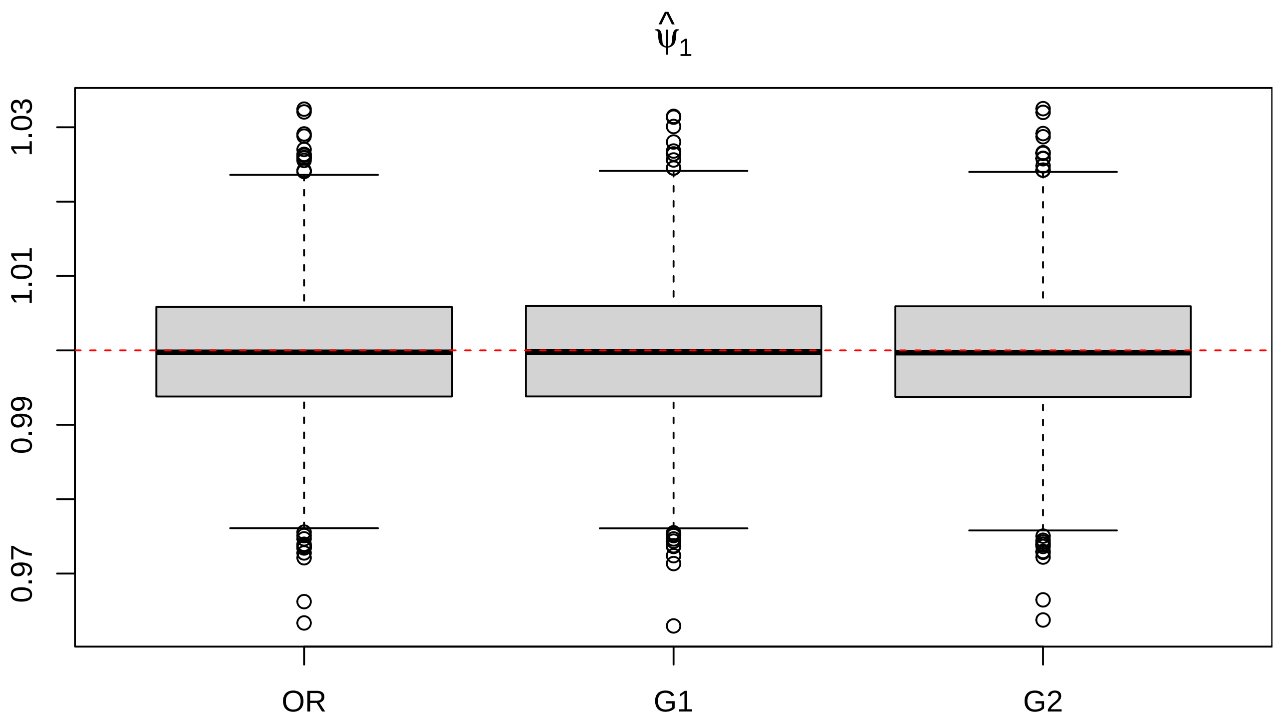
and then fit a model such as

$$m(x,z) = \beta_0 + \psi_0 z + \psi_1 z^2 + \phi_0 b_1(x) + \phi_1 b_2(x)$$

and so on.

We carry out a simulation study of 2000 replicates with $n = 1000$.



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