MATH 556 - ASSIGNMENT 2

To be handed in not later than 5pm, 19th October 2006.
Please hand in during lectures, to Burnside 1235, or to the Mathematics Office Burnside 1005

1 (a) Suppose that X is a continuous rv with pdf f_X and characteristic function (cf) C_X.

(i) Find $C_X(t)$ if the pdf is given by

$$f_X(x) = \exp\{-x - e^{-x}\} \qquad x \in \mathbb{R}.$$

(ii) Find $C_X(t)$ if the pdf is given by

$$f_X(x) = \frac{1}{\cosh(\pi x)} = \frac{2}{e^{-\pi x} + e^{\pi x}} = \sum_{k=0}^{\infty} (-1)^k \exp\{-(2k+1)\pi|x|\} \qquad x \in \mathbb{R}.$$

(iii) Find $f_X(x)$ if the cf is given by

$$C_X(t) = \begin{cases} 1 - |t| & -1 < t < 1 \\ 0 & \text{otherwise} \end{cases}$$

(b) Suppose that random variable Y has cf defined by

$$C_Y(t) = \cos(\theta t) \qquad t \in \mathbb{R}.$$

for some parameter $\theta > 0$. Find the distribution of Y.

16 MARKS

2 Suppose that $X_1, \ldots, X_n$ are independent and identically distributed Cauchy rvs each with pdf

$$f_X(x) = \frac{1}{\pi}\frac{1}{1+x^2} \qquad x \in \mathbb{R}$$

and characteristic function

$$C_X(t) = \exp\{-|t|\} \qquad t \in \mathbb{R}.$$

Let continuous random variable Z_n be defined by

$$Z_n = \frac{1}{\overline{X}} = \frac{n}{\sum\limits_{j=1}^{n} X_j}.$$

Find

$$P[|Z_n| \leq c]$$

for constant $c > 0$.

5 MARKS

3 A probability distribution for rv X is termed *infinitely divisible* if, for all positive integers n, there exists a sequence of independent and identically distributed rvs $Z_{n1}, \ldots, Z_{nn}$ such that X and

$$Z_n = \sum_{j=1}^{n} Z_{nj}$$

have the same distribution, that is, the characteristic function of X can be written

$$C_X(t) = \{C_Z(t)\}^n$$

for some characteristic function C_Z.

Show that the $Gamma(\alpha, \beta)$ distribution is infinitely divisible.

4 MARKS

Note: In completing this assignment, you may quote without proof results from lectures.