MATH 556 - ASSIGNMENT 1

To be handed in not later than 5pm, 28th September 2006.
Please hand in during lectures, to Burnside 1235, or to the Mathematics Office Burnside 1005

1. Suppose X and Y are discrete random variables having joint pmf given by

$$f_{X,Y}(x,y) = c\frac{(x+y)\phi^{x+y}}{x!y!} \qquad x,y \geq 0$$

and zero otherwise, for constant c and parameter $\phi > 0$.

Find expressions for each of the following quantities.

(a) The constant c.

(b) The marginal pmf for X, f_X.

(c) The probability

$$P[X+Y=r]$$

for general $r \geq 0$.

(d) The expectation of X.

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2. Two points A_1 and A_2 are selected independently from the interior of the unit disc $\mathcal{D}$ (the disc centered at the origin, with radius 1), according to the following probability law; a point A is identified using polar coordinate random variables (R,T) (R is the radius, T the angle in radians measured from the x-axis), and the joint pdf of (R,T) is given by

$$f_{R,T}(r,t) = \frac{r}{\pi} \qquad 0 \leq t < 2\pi, 0 < r < 1.$$

and zero otherwise.

Find the probability that the circle centered at A_1 with radius $|A_1A_2|$ (that is, the distance between A_1 and A_2) is contained entirely within $\mathcal{D}$.

Hint: For random point A and set $\mathcal{B}$,

$$P[A \in B] = \int_{\mathcal{B}}\int f_{R,T}(r,t)\; drdt \equiv \int_{\mathcal{B}}\int g(x,y)\; dxdy$$

where the second integral is obtained after changing variables to Cartesian coordinates, for some integrand $g(x,y)$.

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3. A pmf for discrete random variable X taking values on the non-negative integers $\{0,1,2,\ldots\}$ is specified by the countable set of probabilities $\{p_0,p_1,p_2,\ldots\}$, where $P[X=j]=p_j$ for each j. An equivalent specification in terms of the hazard probabilities, $\{h_0,h_1,h_2,\ldots\}$, is also possible, where

$$h_j = h_X(j) = P[X=j \mid X \geq j].$$

Find expressions for

(a) $p_j, j \geq 0$,

(b) the survivor function, $S_X(x) = P[X > x]$.

in terms of $\{h_0,h_1,h_2,\ldots\}$

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