

MATH 523: RESIDUALS

Three types of residual are used for checking the fit of a GLM

- **Pearson residual**

$$r_{Pi} = \frac{y_i - \hat{\mu}_i}{\sqrt{V(\hat{\mu}_i)}}$$

- **Anscombe residual**

$$r_{Ai} = \frac{A(y_i) - A(\hat{\mu}_i)}{\dot{A}(\hat{\mu}_i)\sqrt{V(\hat{\mu}_i)}}$$

where function A is defined by

$$A(t) = \int_{-\infty}^t \frac{ds}{V^{1/3}(s)}$$

The function $A(\cdot)$ is chosen such that, for models in the Exponential/Exponential-Dispersion family, the skewness of the distribution of the transformed variable $A(Y)$ is as small as possible.

- **Deviance Residual**

$$D(\mathbf{y}; \hat{\theta}) = \sum_{i=1}^n D_i(y_i; \hat{\theta}) = \sum_{i=1}^n D_i$$

where

$$D_i = 2w_i \left\{ y_i(\tilde{\theta} - \hat{\theta}_i) - (b(\tilde{\theta}) - b(\hat{\theta}_i)) \right\}$$

Then the deviance residual is

$$r_{Di} = \text{sign}(y_i - \hat{\mu}_i) \sqrt{D_i} = \begin{cases} \sqrt{D_i} & y_i > \hat{\mu}_i \\ -\sqrt{D_i} & y_i < \hat{\mu}_i \end{cases}$$

In the Poisson case

$$\begin{aligned} r_{Pi} &= \frac{y_i - \hat{\mu}_i}{\sqrt{\hat{\mu}_i}} \\ r_{Ai} &= \frac{3(y_i^{2/3} - \hat{\mu}_i^{2/3})}{2\hat{\mu}_i^{1/6}} \\ r_{Di} &= \text{sign}(y_i - \hat{\mu}_i) \left\{ 2(y_i \log \frac{y_i}{\hat{\mu}_i} - y_i + \hat{\mu}_i) \right\}^{1/2} \end{aligned}$$

where $\hat{\mu}_i = \hat{\lambda}_i = \exp \{x_i \beta\}$

```
1 f0<-glm(Y~1, family=poisson)
2 f1<-glm(Y~X, family=poisson)
3
4 rp0<-residuals(f0, type="pearson")
5 rd0<-residuals(f0, type="deviance")
6 fA0<-fitted(f0)
7 rA0<-1.5*(Y^(2/3)-fA0^(2/3))/fA0^(1/6)
8 rp1<-residuals(f1, type="pearson")
9 rd1<-residuals(f1, type="deviance")
10 fA1<-fitted(f1)
11 rA1<-1.5*(Y^(2/3)-fA1^(2/3))/fA1^(1/6)
```

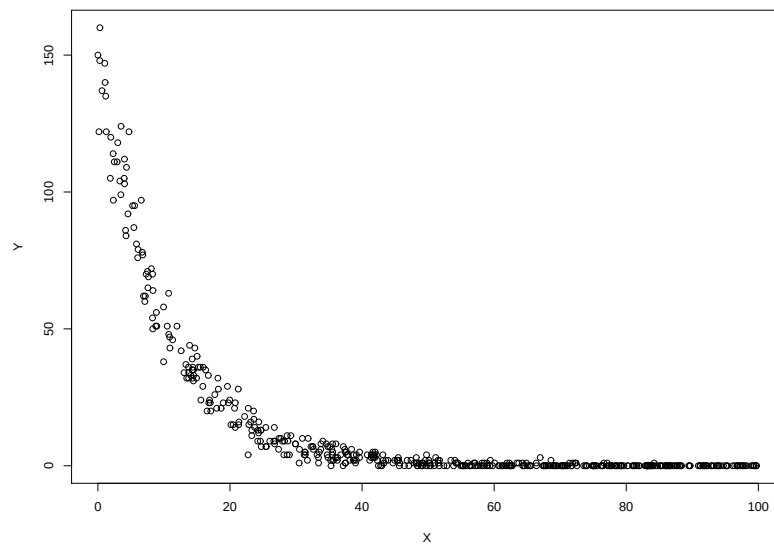


Figure 1: Poisson data Y with single covariate X

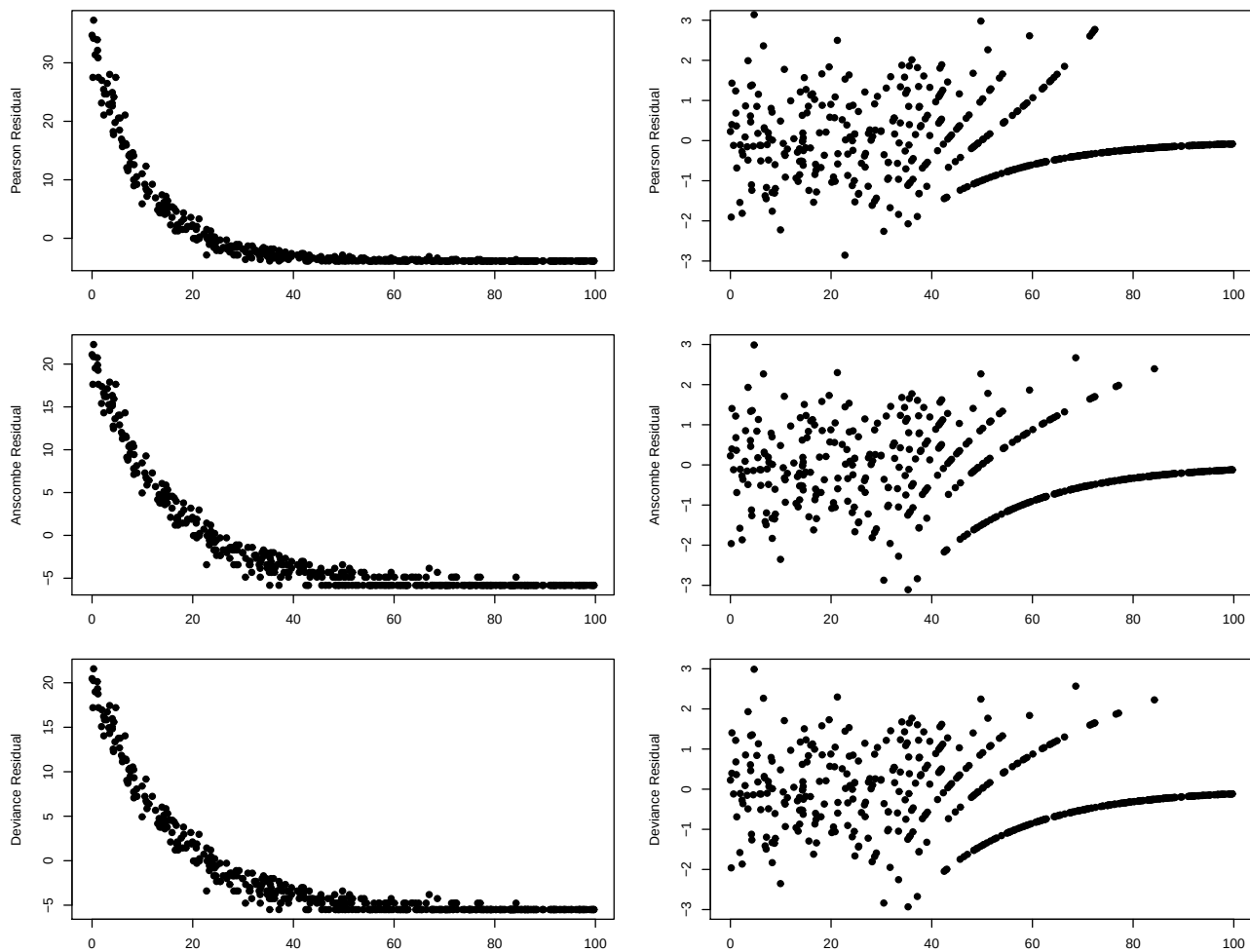


Figure 2: Residuals from fits of null (left column) and single predictor (right column): Pearson (top), Anscombe (middle) and deviance (bottom) residuals.