MATH 204 - ASSIGNMENT 1: SOLUTIONS

1. To compute the ANOVA-F statistic, we first need various quantities that are available from the data given: we have $k = 4$ and $n = 4 \times 35 = 140$, and

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{k}\sum_{j=1}^{n_i} x_{ij} = \frac{1}{140}\sum_{i=1}^{4}(n_i \overline{x}_i) = 11.315$$

$$s_P^2 = \frac{1}{n-k}\sum_{i=1}^{k}(n_i - 1)s_i^2 = 0.749$$

$$\text{SST} = \sum_{i=1}^{k} n_i(\overline{x}_i - \overline{x})^2 = 13.199$$

So therefore

$$\text{MST} = \frac{\text{SST}}{k-1} = 4.400 \qquad \text{MSE} = \frac{\text{SSE}}{n-k} = s_P^2 = 0.749$$

and

$$F = \frac{\text{MST}}{\text{MSE}} = 5.872.$$

We compare this with the Fisher-F$(k-1, n-k) \equiv$ Fisher-F$(3, 136)$ distribution. The $\alpha = 0.05$ critical value, C_R is given by SPSS to be

$$F_\alpha(3, 136) = 2.67$$

so we **reject** the hypothesis of equal treatment means. The McClave and Sincich tables do not tabulate this Fisher-F distribution, but we know that as

$$F_\alpha(3, 136) < F_\alpha(3, 120) = 2.68 < 5.872$$

we can still safely reject the hypothesis of equal means.

8 Marks

Using SPSS we can also compute the p-value as $p = 0.0008$.

2. (a) The SPSS output is attached overleaf. The results indicate that there we can reject the null hypothesis of equal treatment means, and conclude that there **is a significant difference between the mean responses** in the four treatment groups ($F = 8.583$, compared against the Fisher-F$(3, 53)$ distribution, yielding a p-value of 0.000 to three decimal places). Inspection of the descriptive statistics and the boxplots imply that the G+S treatment yields the greatest reduction in body fat mass.

   8 Marks

   (b) Checking the Assumptions:
      (i) Independent samples: this is apparently a completely randomized design, so this assumption is met.
      (ii) Normality of the populations: visual inspection of the boxplot below provides no categorical evidence that the normality assumption is violated, although perhaps there is some skewness evident in the G+S group.
      (iii) Equal Variances: Levene's test (below) implies that the equality of variances is not rejected at the 5 % level (p=0.766)

   4 Marks

SPSS Analysis

Descriptives

Change in Body Fat Mass

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
P+P	14	.064	1.5450	.4129	-.828	.956	-2.6	2.8
P+S	14	-.286	1.2177	.3255	-.989	.417	-3.0	1.2
G+P	13	-1.777	1.6161	.4482	-2.754	-.800	-3.6	1.6
G+S	16	-2.250	1.4679	.3670	-3.032	-1.468	-5.6	-.2
Total	57	-1.091	1.7390	.2303	-1.553	-.630	-5.6	2.8

95% Confidence intervals exclude zero, indicating a significant reduction in body fat mass. Note that these CIs do not correct for multiple testing.

Levene Test of Homogeneity of Variances

Levene Statistic	df1	df2	Sig.
.382	3	53	.766

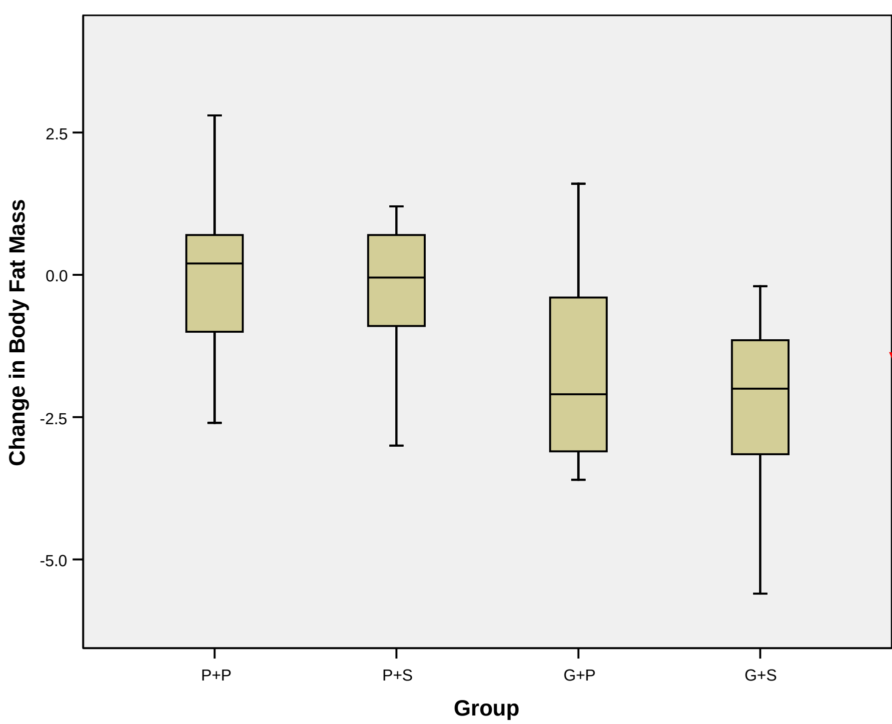
Levene's test indicates that there is no evidence that the subgroup variances are different

ANOVA for Change in Body Fat Mass

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	55.373	3	18.458	8.583	.000
Within Groups	113.972	53	2.150		
Total	169.346	56			

ANOVA-F test indicates that the hypothesis of equal treatment means should be rejected as $p < 0.05$.

BOXPLOT



Boxplot indicates that there is no real evidence to suggest that the subgroup variances are different, and that the assumption that the data are normally distributed seems reasonable, although there may be some indication of skewness in the G+P group.