MATH 680 - ASSIGNMENT 3

To be handed in not later than 5pm EASTERN STANDARD TIME, Friday 17th December 2010.
Projects handed in late will be subject to penalization of up to 100% marks
Please hand in to Burnside 1225, or to the Mathematics Office Burnside 1005.

You may submit by email in pdf format only.

1. The following Bayesian hierarchical model is used to describe the variation of health outcomes of four different types of surgery in six hospitals. Let $i = 1, \ldots, 6$ denote hospital, and $j = 1, \ldots, 4$ denote surgery type. Then, let n_{ij} denote the number of surgeries of type j at hospital i, and y_{ij} denote the number of these surgeries that were successful (that is, surgeries for which the patient was not re-hospitalized in the thirty days after the surgery took place).

$$\begin{aligned}
Y_{ij} &\sim Binomial(n_{ij}, \theta_{ij}) \qquad i = 1, \ldots, 6, j = 1, \ldots, 4 \\
\log\left(\frac{\theta_{ij}}{1-\theta_{ij}}\right) &\sim Normal(\alpha_i + \mu_j, \sigma_j^2) \qquad i = 1, \ldots, 6, j = 1, \ldots, 4 \\
\alpha_i &\sim Normal(0, 10) \qquad i = 1, \ldots, 6 \\
\mu_j &\sim Normal(0, 10) \qquad j = 1, \ldots, 4 \\
1/\sigma_j^2 &\sim Gamma(5, 2.5)
\end{aligned}$$

Construct a Markov chain Monte Carlo (MCMC) algorithm to sample from the joint posterior distribution of all of the unknown parameters, given the observed data and specified prior. Write a report showing exactly how the MCMC is constructed in this case (not the general theory), and the results of an analysis of the following data.

		Surgery Type							
		1		2		3		4	
		y	n	y	n	y	n	y	n
Hospital	1	1	49	7	30	1	29	3	26
	2	11	35	2	17	6	22	1	32
	3	14	26	30	43	15	29	15	31
	4	16	26	23	25	22	34	32	37
	5	29	34	21	25	22	28	16	30
	6	15	30	20	33	13	34	12	39

Verify that the MCMC algorithm has reached convergence using graphical means, and calculate the effective sample size for the posterior samples obtained from your chain, given the actual Monte Carlo sample size you selected.

Explain how these data and your Bayesian results can be used to construct a "league-table" of hospitals ranked on their rates of successful surgery, and construct a suitable Bayesian posterior summary for performing this ranking.

60 Marks

2. The *Burr* distribution is a three-parameter probability distribution on $\mathbb{R}^+$ with cumulative distribution function

$$F_X(x) = 1 - \frac{1}{(1 + \lambda x^\alpha)^\gamma} \qquad x > 0$$

for parameters $\alpha, \lambda, \gamma > 0$. This distribution is useful for modelling right-skewed data.

After specifying a suitable prior distribution, use simulation-based Bayesian inference to produce posterior samples for the unknown parameters, and also a pointwise 95 % credible interval for the survivor function

$$S_X(x) = 1 - F_X(x)$$

derived from the following data

2.83	2.00	5.14	3.83	4.52	1.25	3.57	2.43
2.45	5.98	2.32	3.05	<u>10.00</u>	<u>10.00</u>	2.74	

where underlined observations are **right-censored** (that is, we know that the actual numerical value for that datum is **greater than** the underlined, quoted value).

Give full details of how you implemented your simulation-based inference.

25 Marks

3. Write a review (comprising no more than one page) of the paper ***Monte Carlo Exact Conditional Tests for Log-Linear and Logistic Models***, by J. J. Forster, J. W. McDonald and P. W. F. Smith, *Journal of the Royal Statistical Society. Series B (Methodological)* Vol. 58, No. 2 (1996), pp. 445-453.

Comment in particular on the main contributions and the computational methods used.

15 Marks

This project must be your work alone.

Presentation details:

- Limit your solution to a **maximum** of 20 pages including title page, tables and figures.
- Your project **must be typewritten** in Latex or Word. Format must be
  - 11pt font or greater
  - Pages $8\frac{1}{2}$″ x 11″ (216mm x 279mm) (Letter size)
  - Margins at a minimum of $\frac{3}{4}$″ (18.7 mm)
  - No narrower than single-spaced, with no more than six lines of type per inch.

  A Latex template meeting these requirements is available on request.
- Marks will be deducted for poor presentation.