AIPW Estimation via Regression

For binary treatment Z, the doubly robust AIPW estimator of the average treatment effect (ATE) δ

$$\delta = \mu(1) - \mu(0) = \mathbb{E}[Y(1) - Y(0)]$$

is

$$\widehat{\delta} = \frac{1}{n}\sum_{i=1}^{n}\{\mu(X_i,1) - \mu(X_i,0)\} + \frac{1}{n}\sum_{i=1}^{n}\frac{Z_i}{e(X_i)}(Y_i - \mu(X_i,Z_i)) - \frac{1}{n}\sum_{i=1}^{n}\frac{1-Z_i}{1-e(X_i)}(Y_i - \mu(X_i,Z_i))$$

where $\mu(x,z)$ is the modelled conditional expectation of Y given $Z = z$ and $X = x$. This is the difference of the two estimators of the average potential outcomes (APOs) for treatments 1 and 0 ie for $z = 0, 1$

$$\widetilde{\mu}(z) = \frac{1}{n}\sum_{i=1}^{n}\mu(X_i,z) + \frac{1}{n}\sum_{i=1}^{n}\left(z\frac{Z_i}{e(X_i)} + (1-z)\frac{1-Z_i}{1-e(X_i)}\right)(Y_i - \mu(X_i,Z_i)). \tag{1}$$

For example, for $z = 1$, the corresponding estimating equation is

$$\sum_{i=1}^{n}\left[\mu(X_i,1) + \frac{Z_i}{e(X_i)}(Y_i - \mu(X_i,Z_i)) - \mu(1)\right] = 0. \tag{2}$$

If the true conditional expectation model is $\mu(x,z)$ and the true propensity model is $e(x)$, but that models $m(x,z)$ and $g(x)$ are used, in their place then

$$\begin{aligned}
\mathbb{E}[\widetilde{\mu}(z)] &= \mathbb{E}[m(X_i,z)] + \mathbb{E}\left[\left(z\frac{Z_i}{g(X_i)} + (1-z)\frac{1-Z_i}{1-g(X_i)}\right)(Y_i - m(X_i,Z_i))\right] \\
&= \mathbb{E}[m(X_i,z)] + \mathbb{E}\left[z\frac{Z_i}{g(X_i)}(Y_i - m(X_i,Z_i))\right] + \mathbb{E}\left[(1-z)\frac{1-Z_i}{1-g(X_i)}(Y_i - m(X_i,Z_i))\right] \\
&= \mathbb{E}[m(X_i,z)] + \mathbb{E}\left[z\frac{e(X_i)}{g(X_i)}(Y_i - m(X_i,1))\right] + \mathbb{E}\left[(1-z)\frac{1-e(X_i)}{1-g(X_i)}(Y_i - m(X_i,0))\right] \\
&= \mathbb{E}[m(X_i,z)] + \mathbb{E}\left[z\frac{e(X_i)}{g(X_i)}(\mu(X_i,1) - m(X_i,1))\right] + \mathbb{E}\left[(1-z)\frac{1-e(X_i)}{1-g(X_i)}(\mu(X_i,0) - m(X_i,0))\right].
\end{aligned}$$

- If $m(x,z) \equiv \mu(x,z)$, then the latter two terms are zero, and

$$\mathbb{E}[\widetilde{\mu}(z)] = \mathbb{E}[\mu(X_i,z)].$$

- If $g(x) \equiv e(x)$,

$$\mathbb{E}[\widetilde{\mu}(z)] = \mathbb{E}[m(X_i,z)] + \mathbb{E}\left[z(\mu(X_i,1) - m(X_i,1))\right] + \mathbb{E}\left[(1-z)(\mu(X_i,0) - m(X_i,0))\right] = \mathbb{E}[\widetilde{\mu}(X_i,z)].$$

Hence the estimator is doubly robust.

Regression Approach: The *augmented outcome regression* with

$$\mathbb{E}[Y|Z,X] = \mu(Z,X) + \phi_1\frac{Z}{e(X)} + \phi_0\frac{1-Z}{1-e(X)} = \mu_A(Z,X)$$

say, could be used to estimate the quantity δ using OLS. This is easily seen as the two estimating equations for the parameters ϕ_1 and ϕ_0 are

$$\sum_{i=1}^{n}\frac{Z_i}{e(X_i)}\left(Y_i - \mu(X_i,Z_i) - \phi_1\frac{Z_i}{e(X_i)} - \phi_0\frac{1-Z_i}{1-e(X_i)}\right) = 0$$

$$\sum_{i=1}^{n}\frac{1-Z_i}{1-e(X_i)}\left(Y_i - \mu(X_i,Z_i) - \phi_1\frac{Z_i}{e(X_i)} - \phi_0\frac{1-Z_i}{1-e(X_i)}\right) = 0.$$

or equivalently

$$\sum_{i=1}^{n} \frac{Z_i}{e(X_i)} \left(Y_i - \mu(X_i, Z_i) - \phi_1 \frac{Z_i}{e(X_i)} \right) = 0 \tag{3}$$

$$\sum_{i=1}^{n} \frac{1-Z_i}{1-e(X_i)} \left(Y_i - \mu(X_i, Z_i) - \phi_0 \frac{1-Z_i}{1-e(X_i)} \right) = 0. \tag{4}$$

Thus

$$\widehat{\phi}_1 = \frac{\sum\limits_{i=1}^{n} \frac{Z_i}{e(X_i)} \left(Y_i - \mu(X_i, Z_i) \right)}{\sum\limits_{i=1}^{n} \left(\frac{Z_i}{e(X_i)} \right)^2} \qquad \widehat{\phi}_0 = \frac{\sum\limits_{i=1}^{n} \frac{1-Z_i}{1-e(X_i)} \left(Y_i - \mu(X_i, Z_i) \right)}{\sum\limits_{i=1}^{n} \left(\frac{1-Z_i}{1-e(X_i)} \right)^2}. \tag{5}$$

The corresponding estimators are derived from the fitted values from this model. For example

$$\begin{aligned}
\widetilde{\mu}(1) &= \frac{1}{n} \sum_{i=1}^{n} \mu(X_i, 1) + \widehat{\phi}_1 \frac{1}{n} \sum_{i=1}^{n} \frac{1}{e(X_i)} \\
&= \frac{1}{n} \sum_{i=1}^{n} \mu(X_i, 1) + \frac{\sum\limits_{i=1}^{n} \frac{Z_i}{e(X_i)} \left(Y_i - \mu(X_i, Z_i) \right)}{\sum\limits_{i=1}^{n} \left(\frac{Z_i}{e(X_i)} \right)^2} \frac{1}{n} \sum_{i=1}^{n} \frac{1}{e(X_i)} \\
&= \frac{1}{n} \sum_{i=1}^{n} \mu(X_i, 1) + \frac{\sum\limits_{i=1}^{n} \frac{1}{e(X_i)}}{\sum\limits_{i=1}^{n} \left(\frac{Z_i}{e(X_i)} \right)^2} \frac{1}{n} \sum_{i=1}^{n} \frac{Z_i}{e(X_i)} \left(Y_i - \mu(X_i, Z_i) \right)
\end{aligned} \tag{6}$$

Now,

$$\frac{1}{n} \sum_{i=1}^{n} \frac{1}{e(X_i)} \xrightarrow{p} \mathbb{E}\left[\frac{1}{e(X)} \right] \qquad \frac{1}{n} \sum_{i=1}^{n} \left(\frac{Z_i}{e(X_i)} \right)^2 \xrightarrow{p} \mathbb{E}\left[\left(\frac{Z}{e(X)} \right)^2 \right] = \mathbb{E}\left[\frac{1}{e(X)} \right]$$

so the two estimators in (1) and (6) are asymptotically identical. The doubly robust estimating equation being solved here is the equivalent to (2), that is

$$\sum_{i=1}^{n} \left[\widetilde{\mu}_A(X_i, 1) + \frac{Z_i}{e(X_i)} (Y_i - \widetilde{\mu}_A(X_i, Z_i)) - \mu(1) \right] = 0 \tag{7}$$

where

$$\widetilde{\mu}_A(X_i, 1) = \mu(X_i, 1) + \widehat{\phi}_1 \frac{1}{e(X_i)}.$$

By (3), we have that

$$\sum_{i=1}^{n} \frac{Z_i}{e(X_i)} (Y_i - \widetilde{\mu}_A(X_i, Z_i)) = \sum_{i=1}^{n} \left(Y_i - \mu(X_i, Z_i) - \widehat{\phi}_1 \frac{Z_i}{e(X_i)} \right) \frac{Z_i}{e(X_i)} = 0.$$

The double robustness property relates to the correct specification of $\mu_A(x, z)$ as the conditional expectation of Y given $Z = z$ and $X = x$ (or of $e(x)$ as the propensity model). However, if in fact $\mu(x, z)$ is the correct conditional expectation, then from (5) it is clear that $\widehat{\phi}_1 \xrightarrow{p} 0$ as $n \longrightarrow \infty$.

Simulation study: In the following study we examine the performance of six estimators:

1. $\mu(x,z)$ and $e(x)$ correctly specified, augmented outcome regression estimator;
2. $\mu(x,z)$ and $e(x)$ correctly specified, direct AIPW estimator;
3. $e(x)$ correctly specified, $\mu(x,z)$ incorrectly specified, augmented outcome regression estimator;
4. $e(x)$ correctly specified, $\mu(x,z)$ incorrectly specified, direct AIPW estimator;
5. $\mu(x,z)$ correctly specified, $e(x)$ incorrectly specified, augmented outcome regression estimator;
6. $\mu(x,z)$ correctly specified, $e(x)$ incorrectly specified, direct AIPW estimator;

First we compute the true value of the ATE by simulation using a large sample:

```
library(MASS)
set.seed(23987)
n<-1000000
D<-diag(c(0.25,0.5,0.75))
Mu<-c(1,-2,-1)
Sigma<-D %*% (matrix(c(1,0.9,-0.1,0.9,1,-0.2,-0.1,-0.2,1),3,3) %*% D)
X<-mvrnorm(n,mu=Mu,Sigma)
theta<-2.0
al<-c(6.0,-0.2,0.7,2)
Xal<-cbind(1,X[,1],X[,2],X[,1]*X[,2])
expit<-function(x){return(1/(1+exp(-x)))}
ps.true<-expit(Xal %*% al)
Z<-rbinom(n,1,ps.true)
be<-c(0,theta,-2.0,1.2,0.6)
Xbe<-cbind(1,Z,X[,1],X[,2],X[,1]*Z)
mu.true<- Xbe %*% be
Y<-rnorm(n)*5+mu.true
X0<-cbind(1,0,X[,1],X[,2],X[,1]*0)
X1<-cbind(1,1,X[,1],X[,2],X[,1]*1)
mu0<-X0 %*% be
mu1<-X1 %*% be
ATE.true<-mean(mu1)-mean(mu0)
ATE.true

+ [1] 2.599677

X1<-X[,1]; X2<-X[,2]; X3<-X[,3]
```

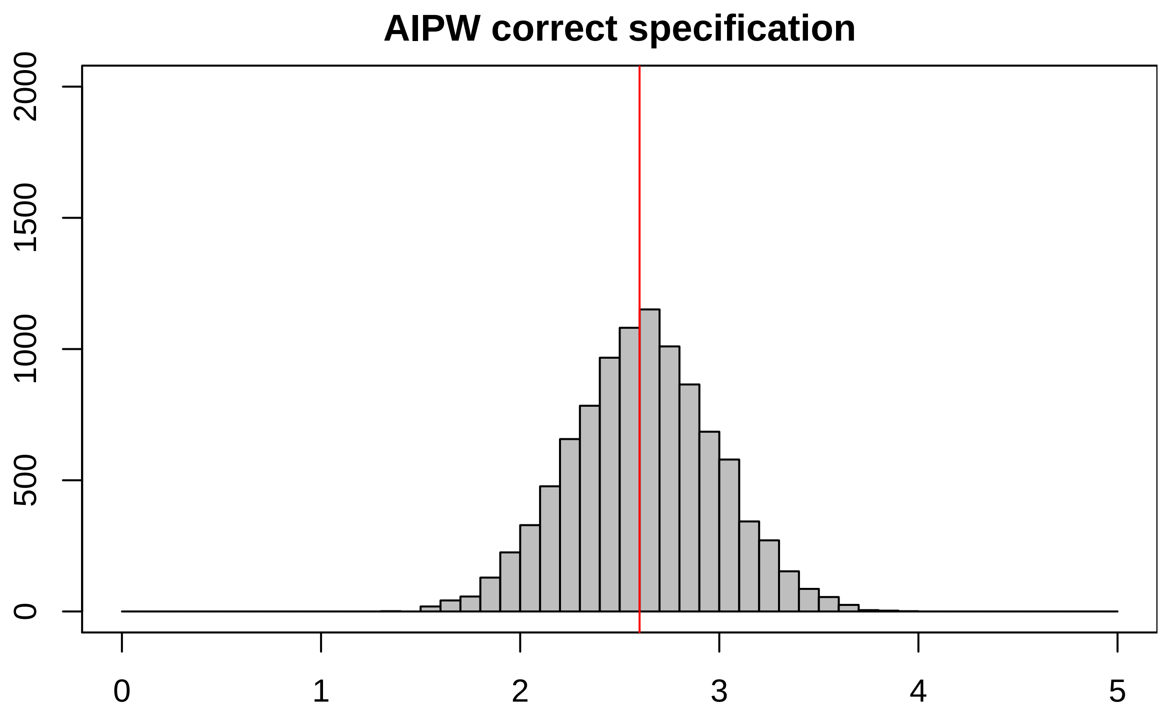
In this simulation, the true ATE value is 2.6.

```
Yoff<-Y-mu.true
C1<-Z/ps.true
phi1.hat<-sum(C1*Yoff)/sum(C1^2)
mu1.hat<-mean(mu1)+phi1.hat*mean(1/ps.true)
C0<-(1-Z)/(1-ps.true)
phi0.hat<-sum(C0*Yoff)/sum(C0^2)
mu0.hat<-mean(mu0)+phi0.hat*mean(1/(1-ps.true))
ATE.hat<-mu1.hat-mu0.hat
Yoff<-Y
C1<-Z/ps.true
phi1.hat<-sum(C1*Yoff)/sum(C1^2)
mu1.hat<-phi1.hat*mean(1/ps.true)
C0<-(1-Z)/(1-ps.true)
phi0.hat<-sum(C0*Yoff)/sum(C0^2)
mu0.hat<-phi0.hat*mean(1/(1-ps.true))
ATE.hat.mis<-mu1.hat-mu0.hat
c(ATE.hat,ATE.hat.mis)

+ [1] 2.616234 2.618082
```

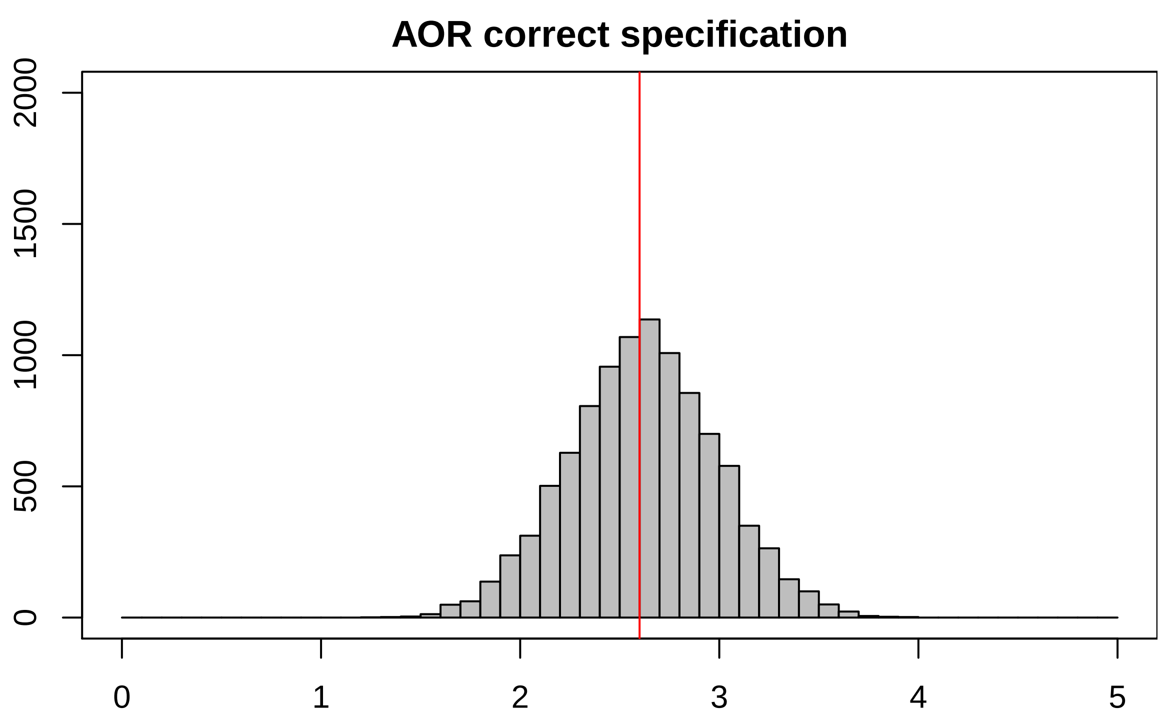
The AOR and the AIPW estimators exhibit apparent consistent estimation; however, they are not numerically identical.

```
nreps<-10000
res.mat<-matrix(0,nrow=nreps,ncol=6)
nsamp<-1000
for(irep in 1:nreps){
        isub<-sample(1:n,size=nsamp,rep=T)
        Yoff<-Y[isub]-mu.true[isub]
        W1.sub<-Z[isub]/ps.true[isub]
        W0.sub<-(1-Z[isub])/(1-ps.true[isub])
        mu.sub<-mu.true[isub]
        Z.sub<-Z[isub]
        mu1.sub<-mu1[isub]
        mu0.sub<-mu0[isub]

    #AIPW estimator, correct specification
        ATE.direct<-mean(mu1.sub)-mean(mu0.sub)+mean(W1.sub*Yoff)-mean(W0.sub*Yoff)

    #AOR estimator, correct specification
        phi1.hat<-sum(W1.sub*Yoff)/sum(W1.sub^2)
        mu1.hat<-mean(mu1.sub)+phi1.hat*mean(1/ps.true[isub])
        phi0.hat<-sum(W0.sub*Yoff)/sum(W0.sub^2)
        mu0.hat<-mean(mu0.sub)+phi0.hat*mean(1/(1-ps.true[isub]))
        ATE.hat<-mu1.hat-mu0.hat

        #Incorrect mean model
    Yoff<-Y[isub]

    #AIPW estimator, incorrect mean specification
        ATE.direct.mis<-mean(W1.sub*Yoff)-mean(W0.sub*Yoff)

    #AOR estimator, incorrect mean specification
        phi1.hat<-sum(W1.sub*Yoff)/sum(W1.sub^2)
        mu1.hat<-phi1.hat*mean(1/ps.true[isub])
        phi0.hat<-sum(W0.sub*Yoff)/sum(W0.sub^2)
        mu0.hat<-phi0.hat*mean(1/(1-ps.true[isub]))
        ATE.hat.mis<-mu1.hat-mu0.hat

        #Incorrect PS model
    Yoff<-Y[isub]-mu.true[isub]

    ps.mis<-fitted(glm(Z[isub]~X1[isub],family=binomial))
        W1.sub<-Z[isub]/ps.mis
        W0.sub<-(1-Z[isub])/(1-ps.mis)

    #AIPW estimator incorrect ps specification
        ATE.direct.ps.mis<-mean(mu1.sub)-mean(mu0.sub)+mean(W1.sub*Yoff)-mean(W0.sub*Yoff)

    #AOR estimator, incorrect ps specification
        phi1.hat<-sum(W1.sub*Yoff)/sum(W1.sub^2)
        mu1.hat<-mean(mu1.sub)+phi1.hat*mean(1/ps.mis)
        phi0.hat<-sum(W0.sub*Yoff)/sum(W0.sub^2)
        mu0.hat<-mean(mu0.sub)+phi0.hat*mean(1/(1-ps.mis))
        ATE.ps.mis<-mu1.hat-mu0.hat

        res.mat[irep,]<-c(ATE.direct,ATE.hat,ATE.direct.mis,ATE.hat.mis,ATE.direct.ps.mis,ATE.ps.mis)
}
```

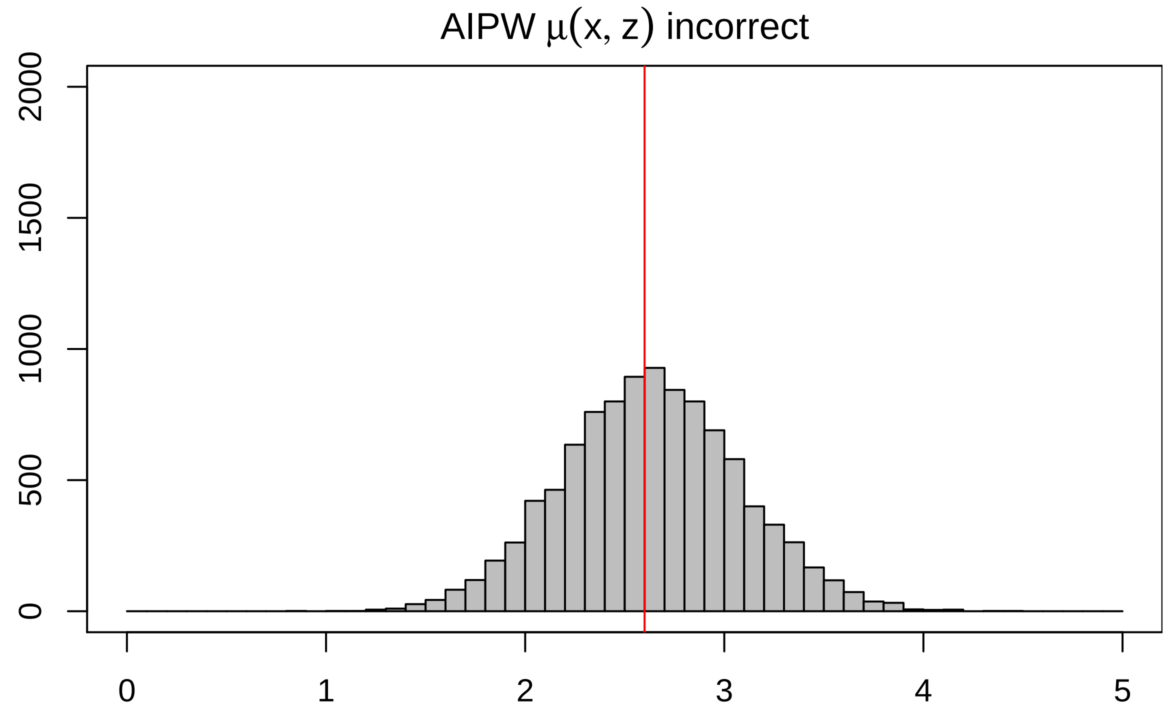
```
br<-seq(0.0,5.0,by=0.1);par(mar=c(2,2,2,0));res<-res.mat[,1];res<-res[res>0 & res < 5]
hist(res,ylim=range(0,2000),breaks=br,main='AIPW correct specification',col='gray',xlab='ATE')
box();abline(v=ATE.true,col='red',lwd=1)
```



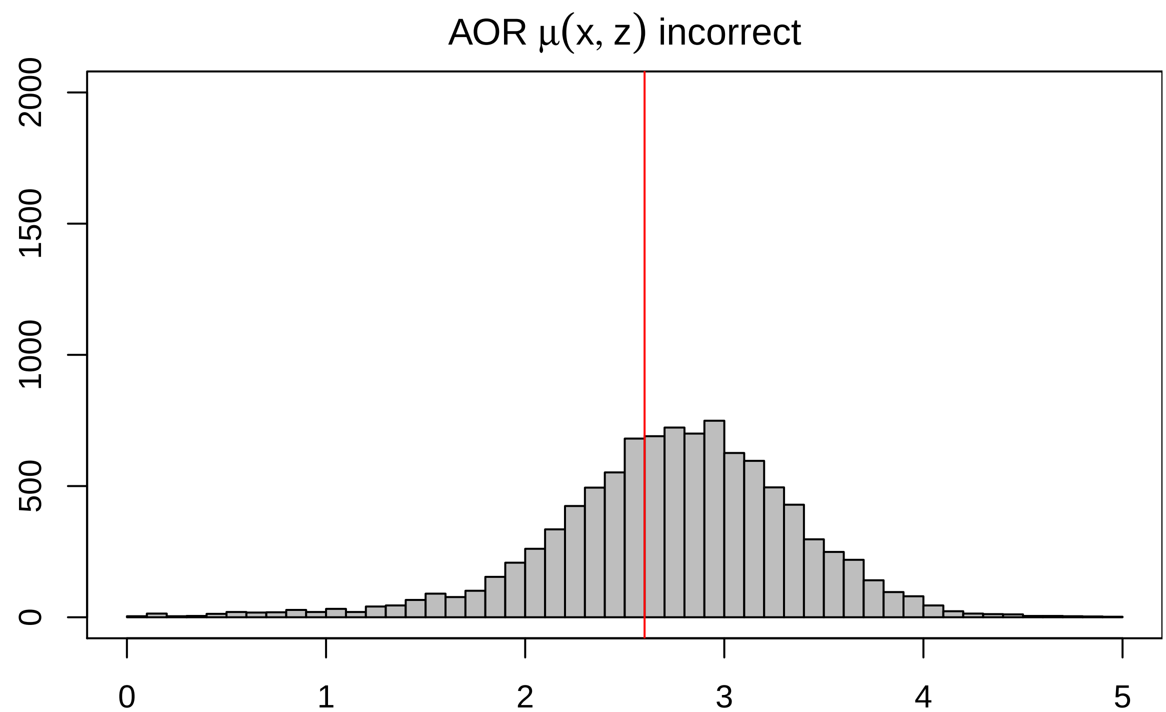

```
br<-seq(0.0,5.0,by=0.1);par(mar=c(2,2,2,0));res<-res.mat[,2];res<-res[res>0 & res < 5]

hist(res,ylim=range(0,2000),breaks=br,main='AOR correct specification',col='gray',xlab='ATE')
box();abline(v=ATE.true,col='red',lwd=1)
```



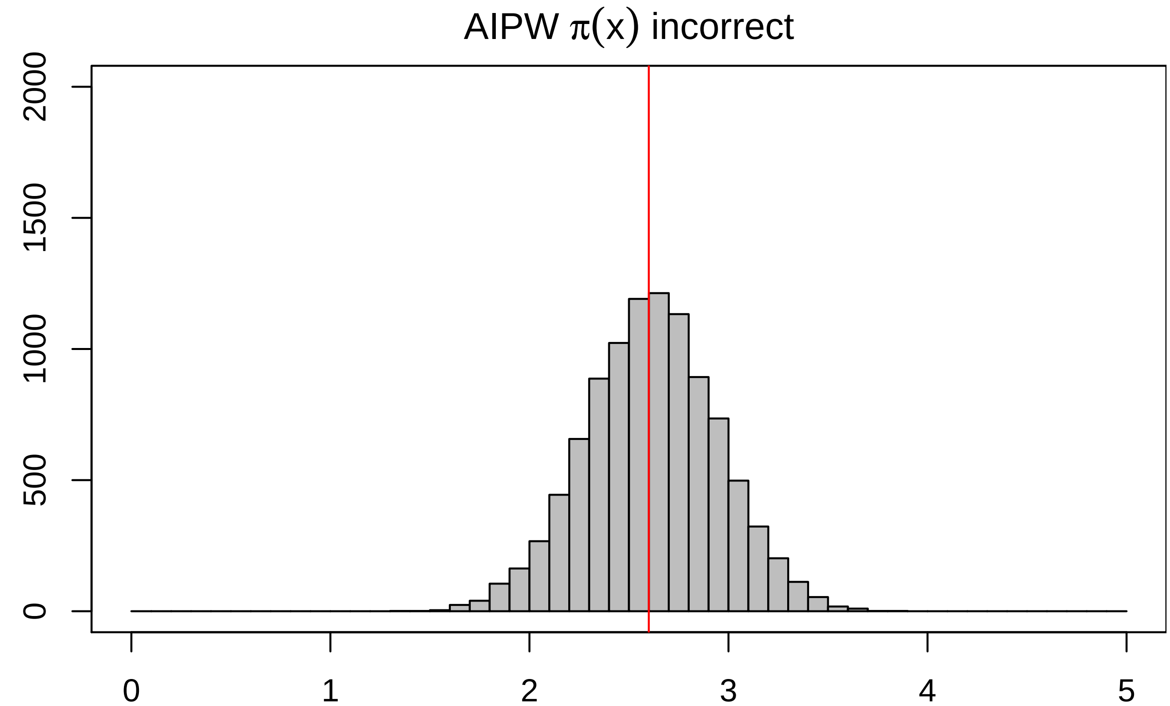
```r
br<-seq(0.0,5.0,by=0.1);par(mar=c(2,2,2,0));res<-res.mat[,3];res<-res[res>0 & res < 5]
hist(res,ylim=range(0,2000),breaks=br,
     main=expression(paste('AIPW ',mu(x,z),' incorrect')),col='gray',xlab='ATE')
box();abline(v=ATE.true,col='red',lwd=1)
```



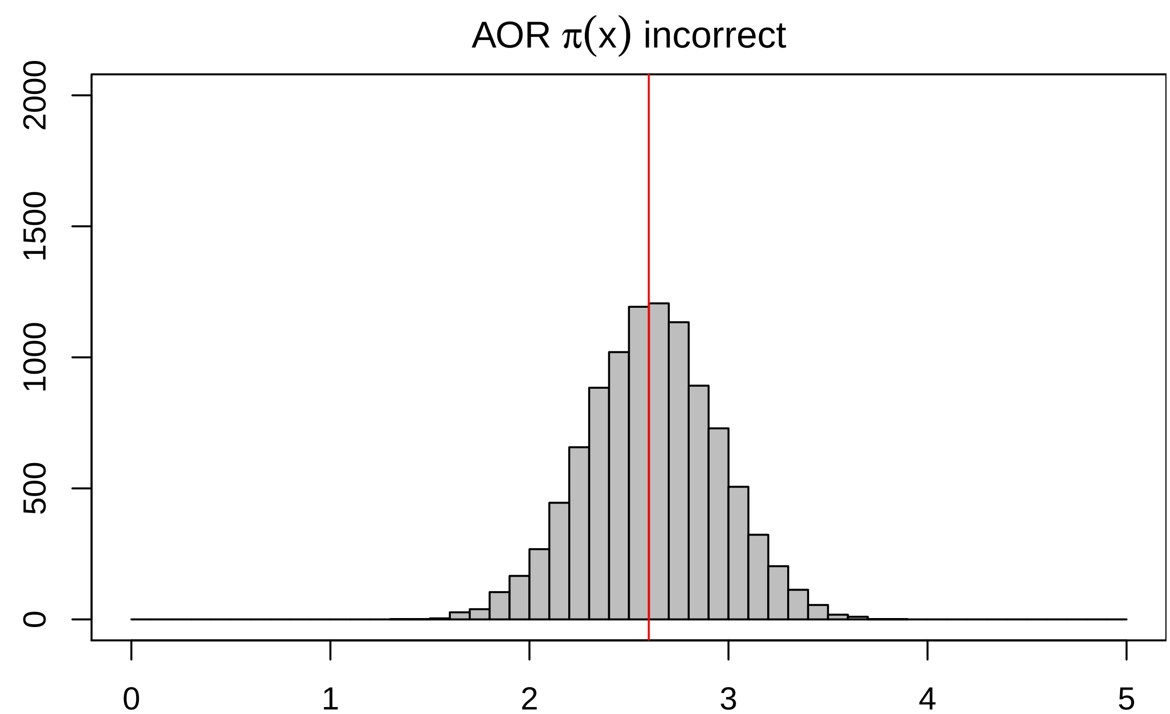

```r
br<-seq(0.0,5.0,by=0.1);par(mar=c(2,2,2,0));res<-res.mat[,4];res<-res[res>0 & res < 5]
hist(res,ylim=range(0,2000),breaks=br,
     main=expression(paste('AOR ',mu(x,z),' incorrect')),col='gray',xlab='ATE')
box();abline(v=ATE.true,col='red',lwd=1)
```



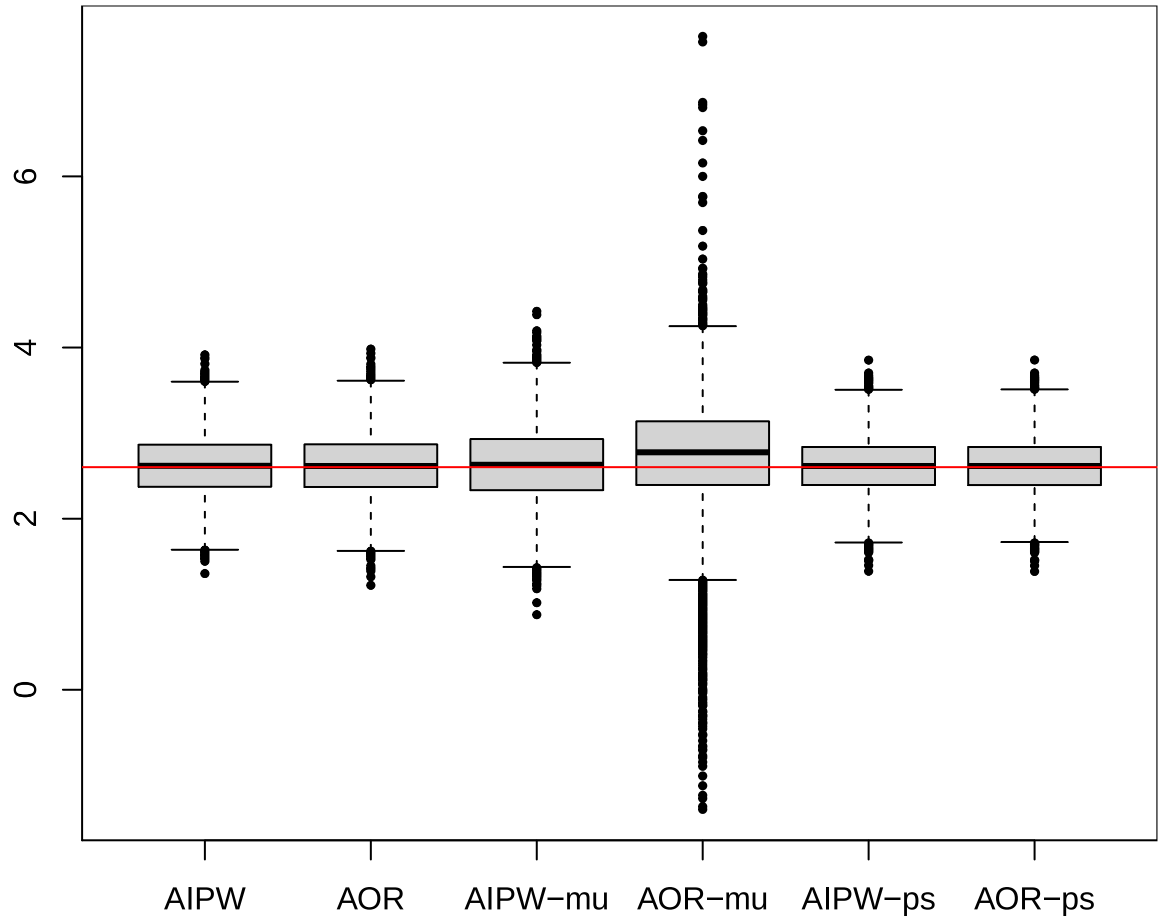
```r
br<-seq(0.0,5.0,by=0.1);par(mar=c(2,2,2,0));res<-res.mat[,5];res<-res[res>0 & res < 5]
hist(res,ylim=range(0,2000),breaks=br,
     main=expression(paste('AIPW ',pi(x),' incorrect')),col='gray',xlab='ATE')
box();abline(v=ATE.true,col='red',lwd=1)
```




```r
br<-seq(0.0,5.0,by=0.1);par(mar=c(2,2,2,0));res<-res.mat[,6];res<-res[res>0 & res < 5]
hist(res,ylim=range(0,2000),breaks=br,
     main=expression(paste('AOR ',pi(x),' incorrect')),col='gray',xlab='ATE')
box();abline(v=ATE.true,col='red',lwd=1)
```



```
par(mar=c(2,2,0,0))
nvec<-c("AIPW","AOR","AIPW-mu","AOR-mu","AIPW-ps","AOR-ps")
boxplot(res.mat,names=nvec,pch=19,cex=0.5);abline(h=ATE.true,col='red',lwd=1)
```




```
(apply(res.mat,2,mean)-ATE.true)*sqrt(nsamp)   #Bias

+ [1] 0.6311402 0.5702369 1.0822978 4.2138020 0.4782042 0.4786384

apply(res.mat,2,var)*nsamp                     #Variance

+ [1] 134.4380 137.0302 198.1056 446.8012 109.8144 110.3034

apply((res.mat-ATE.true)^2,2,mean)*nsamp       #MSE

+ [1] 134.8229 137.3417 199.2571 464.5126 110.0321 110.5214
```

The second Bang and Robins approach proposes taking $\phi_0 = -\phi_1$, and using the AOR model

$$\mathbb{E}[Y|Z,X] = \mu(Z,X) + \phi\left(\frac{Z}{e(X)} - \frac{1-Z}{1-e(X)}\right)$$

to produce the ATE estimator

$$\frac{1}{n}\sum_{i=1}^{n}\{\mu(X_i,1) - \mu(X_i,0)\} + \widehat{\phi}\sum_{i=1}^{n}\left(\frac{1}{e(X_i)} + \frac{1}{1-e(X_i)}\right).$$

By elementary calculations we have that

$$\widehat{\phi} = \frac{\sum_{i=1}^{n} \left(\frac{Z_i}{e(X_i)} - \frac{1-Z_i}{1-e(X_i)} \right) (Y_i - \mu(X_i, Z_i))}{\sum_{i=1}^{n} \left(\frac{Z_i}{e(X_i)} - \frac{1-Z_i}{1-e(X_i)} \right)^2}$$

so that

$$\begin{aligned}
\widehat{\delta} &= \frac{1}{n} \sum_{i=1}^{n} \{\mu(X_i, 1) - \mu(X_i, 0)\} + \frac{1}{n} \sum_{i=1}^{n} \left(\frac{1}{e(X_i)} + \frac{1}{1-e(X_i)} \right) \frac{\sum_{i=1}^{n} \left(\frac{Z_i}{e(X_i)} - \frac{1-Z_i}{1-e(X_i)} \right) (Y_i - \mu(X_i, Z_i))}{\sum_{i=1}^{n} \left(\frac{Z_i}{e(X_i)} - \frac{1-Z_i}{1-e(X_i)} \right)^2} \\
&= \frac{1}{n} \sum_{i=1}^{n} \{\mu(X_i, 1) - \mu(X_i, 0)\} + \frac{\sum_{i=1}^{n} \left(\frac{1}{e(X_i)} + \frac{1}{1-e(X_i)} \right)}{\sum_{i=1}^{n} \left(\frac{Z_i}{e(X_i)} - \frac{1-Z_i}{1-e(X_i)} \right)^2} \frac{1}{n} \sum_{i=1}^{n} \left(\frac{Z_i}{e(X_i)} - \frac{1-Z_i}{1-e(X_i)} \right) (Y_i - \mu(X_i, Z_i)). \quad (8)
\end{aligned}$$

As $n \longrightarrow \infty$ we have that

$$\frac{\sum_{i=1}^{n} \left(\frac{1}{e(X_i)} + \frac{1}{1-e(X_i)} \right)}{\sum_{i=1}^{n} \left(\frac{Z_i}{e(X_i)} - \frac{1-Z_i}{1-e(X_i)} \right)^2} \xrightarrow{p} \frac{\mathbb{E}\left[\frac{1}{e(X)} + \frac{1}{1-e(X)} \right]}{\mathbb{E}\left[\left(\frac{Z}{e(X)} \right)^2 + \left(\frac{1-Z}{1-e(X)} \right)^2 \right]} = 1$$

so that $\widehat{\delta}$ is asymptotically identical to

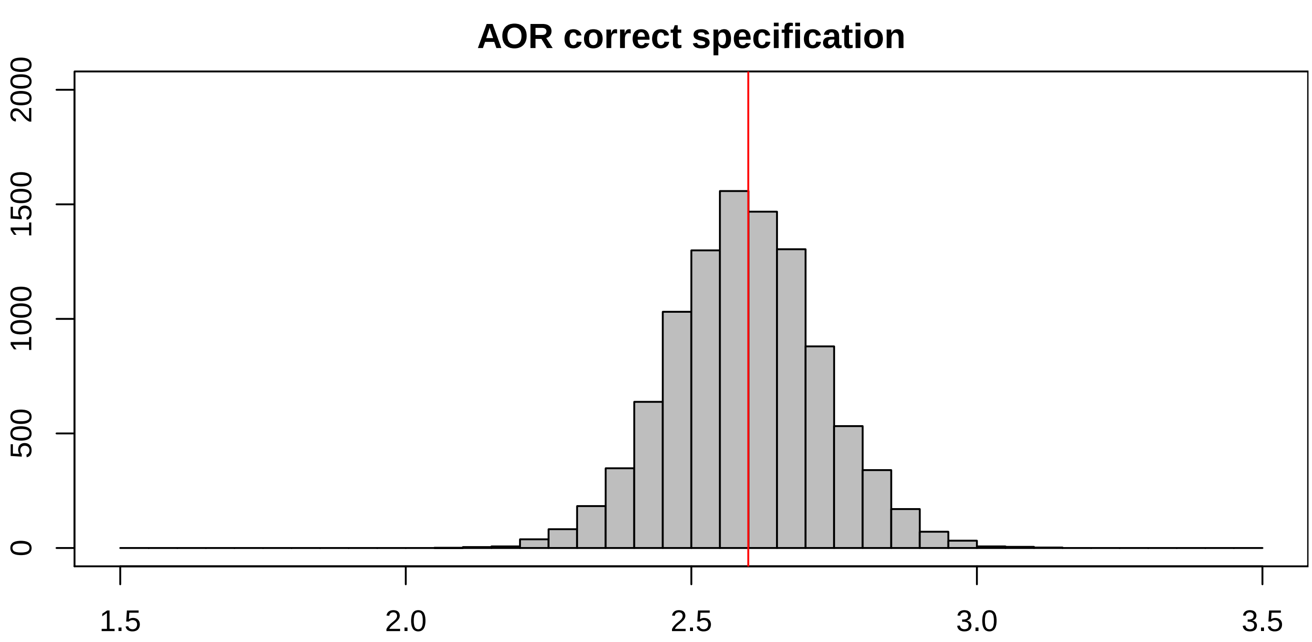
$$\frac{1}{n} \sum_{i=1}^{n} \{\mu(X_i, 1) - \mu(X_i, 0)\} + \frac{1}{n} \sum_{i=1}^{n} \left(\frac{Z_i}{e(X_i)} - \frac{1-Z_i}{1-e(X_i)} \right) (Y_i - \mu(X_i, Z_i))$$

that is, the AIPW estimator.

```
nreps<-10000
res.mat2<-matrix(0,nrow=nreps,ncol=3)
phi.ests<-matrix(0,nrow=nreps,ncol=3)
nsamp<-1000
for(irep in 1:nreps){

        isub<-sample(1:n,size=nsamp,rep=T)

        Yoff<-Y[isub]-mu.true[isub]
        C.sub<-(Z[isub]/ps.true[isub]-(1-Z[isub])/(1-ps.true[isub]))

        mu.sub<-mu.true[isub]
        Z.sub<-Z[isub]
        mu1.sub<-mu1[isub]
        mu0.sub<-mu0[isub]

    #AOR estimator, correct specification
        phi.hat<-sum(C.sub*Yoff)/sum(C.sub^2)
        mu1.hat<-mean(mu1.sub)+phi.hat*mean(1/ps.true[isub])
        mu0.hat<-mean(mu0.sub)+phi.hat*mean(1/(1-ps.true[isub]))
        ATE.hat<-mu1.hat-mu0.hat

    phi.ests[irep,1]<-phi.hat

        #Incorrect mean model
```

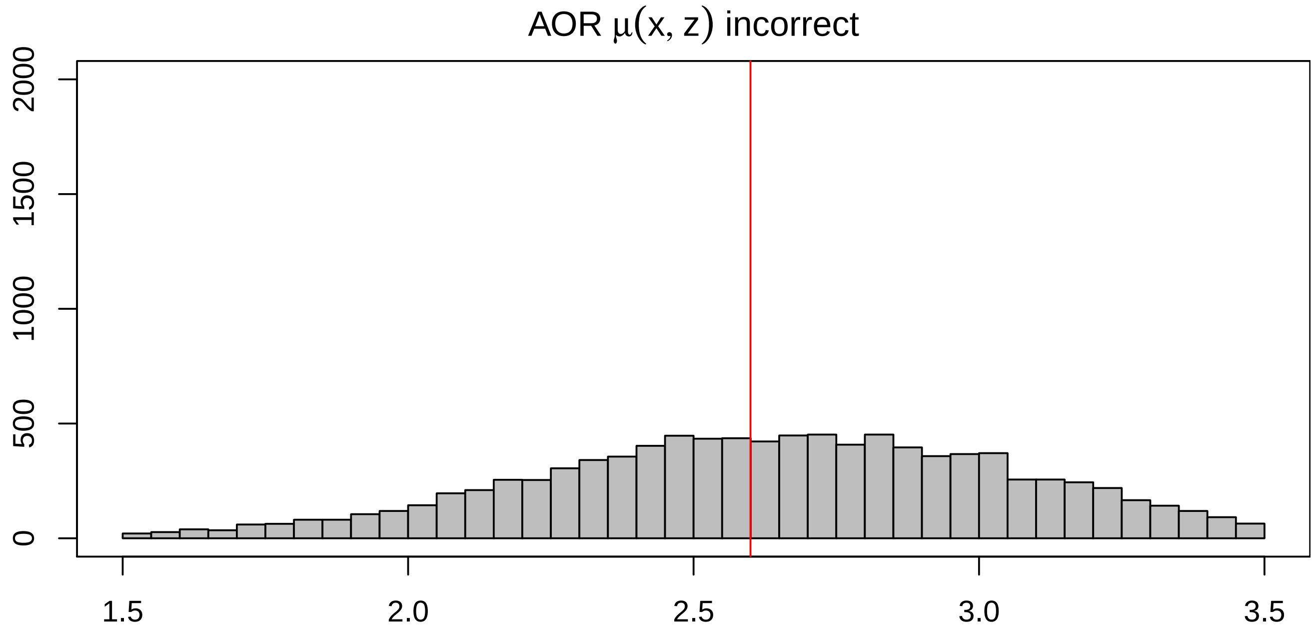
```
    Yoff<-Y[isub]

    #AOR estimator, incorrect mean specification
        phi.hat<-sum(C.sub*Yoff)/sum(C.sub^2)
        mu1.hat<-phi.hat*mean(1/ps.true[isub])
        mu0.hat<--phi.hat*mean(1/(1-ps.true[isub]))
        ATE.hat.mis<-mu1.hat-mu0.hat

    phi.ests[irep,2]<-phi.hat

        #Incorrect PS model
        Yoff<-Y[isub]-mu.true[isub]
    ps.mis<-fitted(glm(Z[isub]~X1[isub],family=binomial))

        C.sub<-Z[isub]/ps.mis-(1-Z[isub])/(1-ps.mis)

    #AOR estimator, incorrect ps specification
        phi.hat<-sum(C.sub*Yoff)/sum(C.sub^2)
        mu1.hat<-mean(mu1.sub)+phi.hat*mean(1/ps.mis)
        mu0.hat<-mean(mu0.sub)-phi.hat*mean(1/(1-ps.mis))
        ATE.ps.mis<-mu1.hat-mu0.hat

    phi.ests[irep,3]<-phi.hat

        res.mat2[irep,]<-c(ATE.hat,ATE.hat.mis,ATE.ps.mis)
}
```

```
br<-seq(1.5,3.5,by=0.05);par(mar=c(3,2,2,0))
res<-res.mat2[,1];res<-res[res>1.5 & res < 3.5]
hist(res,ylim=range(0,2000),breaks=br,
     main='AOR correct specification',col='gray',xlab='ATE')
box();abline(v=ATE.true,col='red',lwd=1)
```

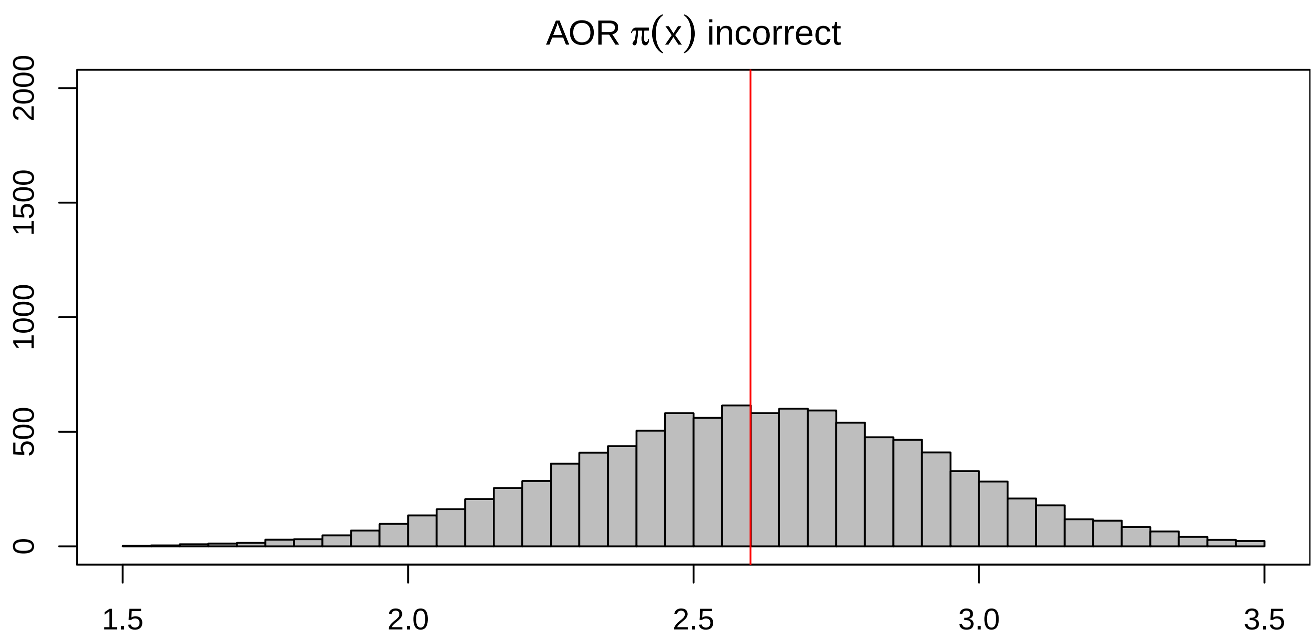



```
res<-res.mat2[,2];res<-res[res>1.5 & res < 3.5]
hist(res,ylim=range(0,2000),breaks=br,
     main=expression(paste('AOR ',mu(x,z),' incorrect')),col='gray',xlab='ATE')
```

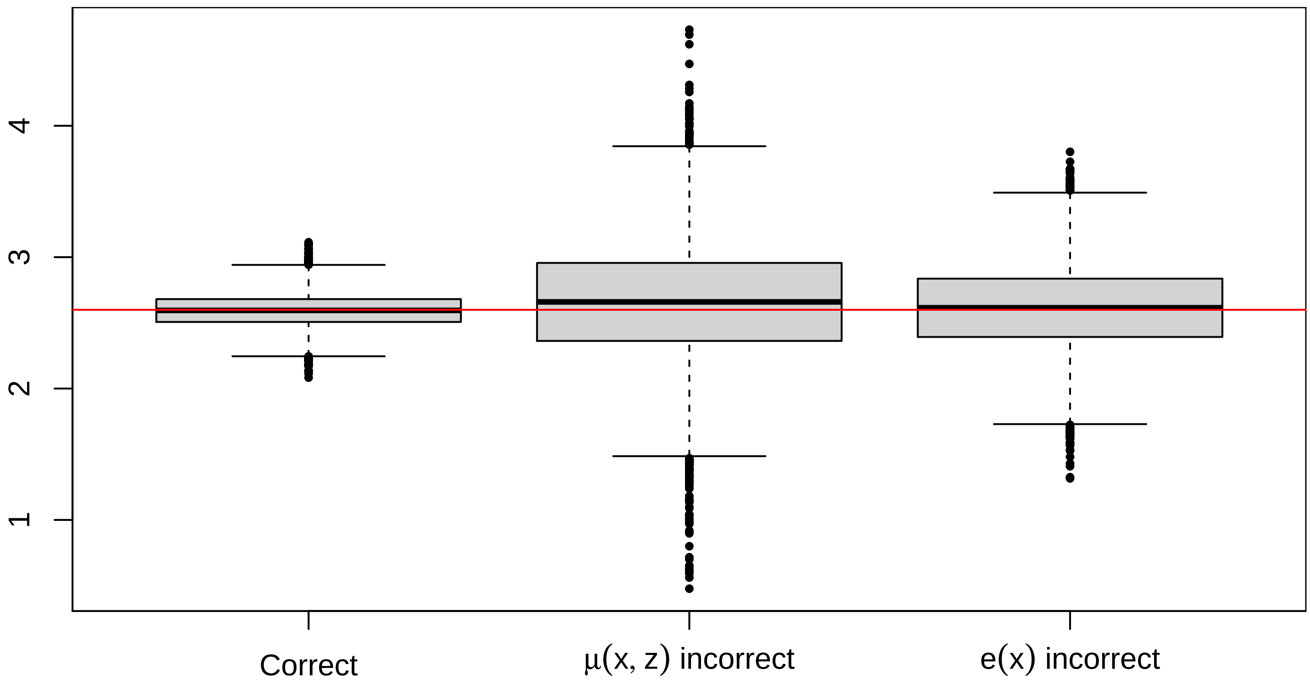
```
box();abline(v=ATE.true,col='red',lwd=1)
```




```
res<-res.mat2[,3];res<-res[res>1.5 & res < 3.5]
hist(res,ylim=range(0,2000),breaks=br,
     main=expression(paste('AOR ',pi(x),' incorrect')),col='gray',xlab='ATE')
box();abline(v=ATE.true,col='red',lwd=1)
```




```
par(mar=c(2,2,0,0))
nvec<-c("Correct",expression(paste(mu(x,z),' incorrect')),expression(paste(e(x),' incorrect')))
boxplot(res.mat2,names=nvec,pch=19,cex=0.5);abline(h=ATE.true,col='red',lwd=1)
```




```
apply(res.mat2,2,var)*nsamp

+ [1]  17.24703 206.61754 108.69031

apply((res.mat2-ATE.true)^2,2,mean)*nsamp

+ [1]  17.2676 209.2598 108.8445
```

```
par(mar=c(2,2,2,0))
nvec<-c("Correct",expression(paste(mu(x,z),' incorrect')),expression(paste(e(x),' incorrect')))
boxplot(phi.ests,names=nvec,pch=19,cex=0.5);abline(h=ATE.true,col='red',lwd=1)
abline(h=0,col='red',lty=2)
title(expression(paste('Distribution of ',hat(phi),' under the correct and incorrect specifications')))
```