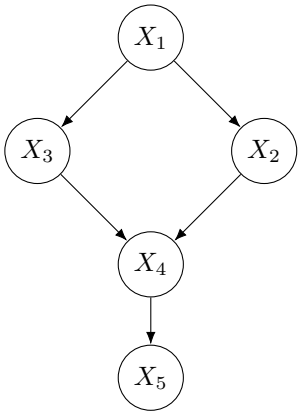
d -SEPARATION EXAMPLE

In the graph

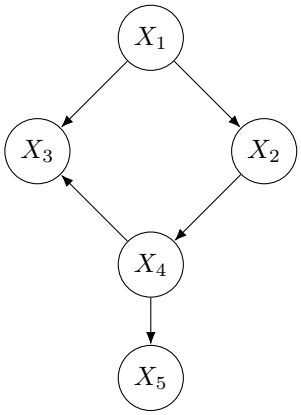


we have that there are two paths connecting X_2 and X_3, namely

$$(X_2, X_1, X_3) \qquad (X_2, X_4, X_3)$$

- the set $S \equiv \{X_1\}$ d-separates X_2 and X_3 as conditioning on X_1 blocks the path (X_2, X_1, X_3); we do not need to consider the second path here, as that is already blocked by the collider X_4;
- the set $S \equiv \{X_1, X_4\}$ does not d-separate X_2 and X_3 as conditioning on the collider X_4 opens the path;
- the set $S \equiv \{X_1, X_5\}$ does not d-separate X_2 and X_3 as conditioning on X_5, a descendant of X_4, opens the path at the collider.

However, in the graph



the same two paths are present, but

- the set $S \equiv \{X_1\}$ does not d-separate X_2 and X_3 as conditioning on X_1 blocks the path (X_2, X_1, X_3); but there is an open path through X_4;
- the set $S \equiv \{X_1, X_4\}$ d-separates X_2 and X_3 as conditioning on X_1 and X_4 blocks both paths;
- the set $S \equiv \{X_1, X_5\}$ does not d-separate X_2 and X_3 as the second path remains open at X_4.

In general, when considering d-separation of two variables X and Y by a set of variables, S, we need to consider **all paths** between X and Y, not merely those involving S - however, in the first example X_4 is a collider so we do not need to consider the second path when taking $S \equiv \{X_1\}$.