

## BANG & ROBINS FOR AUGMENTED IPW

For binary treatment  $Z$ , the doubly robust AIPW estimator of the ATE  $\delta$

$$\delta = \mu(1) - \mu(0) = \mathbb{E}[Y(1) - Y(0)]$$

is

$$\hat{\delta} = \frac{1}{n} \sum_{i=1}^n \{\mu(1, X_i) - \mu(0, X_i)\} + \frac{1}{n} \sum_{i=1}^n \frac{Z_i}{\pi(X_i)} (Y_i - \mu(Z_i, X_i)) - \frac{1}{n} \sum_{i=1}^n \frac{1 - Z_i}{1 - \pi(X_i)} (Y_i - \mu(Z_i, X_i))$$

where  $\mu(z, x)$  is the modelled conditional expectation of  $Y$  given  $Z = z$  and  $X = x$ . This is the difference of the two estimators of the APOs for treatments 1 and 0 ie for  $z = 0, 1$

$$\hat{\mu}(z) = \frac{1}{n} \sum_{i=1}^n \mu(z, X_i) + \frac{1}{n} \sum_{i=1}^n \left( z \frac{Z_i}{\pi(X_i)} + (1 - z) \frac{1 - Z_i}{1 - \pi(X_i)} \right) (Y_i - \mu(Z_i, X_i)). \quad (1)$$

For example, for  $z = 1$ , the corresponding estimating equation is

$$\sum_{i=1}^n \left[ \mu(1, X_i) + \frac{Z_i}{\pi(X_i)} (Y_i - \mu(Z_i, X_i)) - \mu(1) \right] = 0. \quad (2)$$

If the true conditional expectation model is  $\tilde{\mu}(z, x)$ , and the true propensity model is  $\tilde{\pi}(x)$ , it is easy to see that this estimator is doubly robust as

$$\begin{aligned} \mathbb{E}[\hat{\mu}(z)] &= \mathbb{E}[\mu(z, X_i)] + \mathbb{E} \left[ \left( z \frac{Z_i}{\pi(X_i)} + (1 - z) \frac{1 - Z_i}{1 - \pi(X_i)} \right) (Y_i - \mu(Z_i, X_i)) \right] \\ &= \mathbb{E}[\mu(z, X_i)] + \mathbb{E} \left[ z \frac{Z_i}{\pi(X_i)} (Y_i - \mu(Z_i, X_i)) \right] + \mathbb{E} \left[ (1 - z) \frac{1 - Z_i}{1 - \pi(X_i)} (Y_i - \mu(Z_i, X_i)) \right] \\ &= \mathbb{E}[\mu(z, X_i)] + \mathbb{E} \left[ z \frac{\tilde{\pi}(X_i)}{\pi(X_i)} (Y_i - \mu(1, X_i)) \right] + \mathbb{E} \left[ (1 - z) \frac{1 - \tilde{\pi}(X_i)}{1 - \pi(X_i)} (Y_i - \mu(0, X_i)) \right] \\ &= \mathbb{E}[\mu(z, X_i)] + \mathbb{E} \left[ z \frac{\tilde{\pi}(X_i)}{\pi(X_i)} (\tilde{\mu}(1, X_i) - \mu(1, X_i)) \right] + \mathbb{E} \left[ (1 - z) \frac{1 - \tilde{\pi}(X_i)}{1 - \pi(X_i)} (\tilde{\mu}(0, X_i) - \mu(0, X_i)) \right]. \end{aligned}$$

If  $\mu(z, x) \equiv \tilde{\mu}(z, x)$ , then the latter two terms are zero; if  $\pi(x) \equiv \tilde{\pi}(x)$ ,

$$\mathbb{E}[\hat{\mu}(z)] = \mathbb{E}[\mu(z, X_i)] + \mathbb{E} [z(\tilde{\mu}(1, X_i) - \mu(1, X_i))] + \mathbb{E} [(1 - z)(\tilde{\mu}(0, X_i) - \mu(0, X_i))] = \mathbb{E}[\tilde{\mu}(z, X_i)].$$

The Scharfstein et al./Bang and Robins results shows that the augmented outcome regression with

$$\mathbb{E}[Y|Z, X] = \mu(Z, X) + \phi_1 \frac{Z}{\pi(X)} + \phi_0 \frac{1 - Z}{1 - \pi(X)} = \mu_A(Z, X)$$

say, could be used to estimate the quantity  $\delta$  using OLS. This is easily seen as the two estimating equations for the parameters  $\phi_1$  and  $\phi_0$  are

$$\begin{aligned} \sum_{i=1}^n \left( Y_i - \mu(Z_i, X_i) - \phi_1 \frac{Z_i}{\pi(X_i)} - \phi_0 \frac{1 - Z_i}{1 - \pi(X_i)} \right) \frac{Z_i}{\pi(X_i)} &= 0 \\ \sum_{i=1}^n \left( Y_i - \mu(Z_i, X_i) - \phi_1 \frac{Z_i}{\pi(X_i)} - \phi_0 \frac{1 - Z_i}{1 - \pi(X_i)} \right) \frac{1 - Z_i}{1 - \pi(X_i)} &= 0. \end{aligned}$$

or equivalently

$$\sum_{i=1}^n \left( Y_i - \mu(Z_i, X_i) - \phi_1 \frac{Z_i}{\pi(X_i)} \right) \frac{Z_i}{\pi(X_i)} = 0 \quad (3)$$

$$\sum_{i=1}^n \left( Y_i - \mu(Z_i, X_i) - \phi_0 \frac{1 - Z_i}{1 - \pi(X_i)} \right) \frac{1 - Z_i}{1 - \pi(X_i)} = 0. \quad (4)$$

Thus

$$\hat{\phi}_1 = \frac{\sum_{i=1}^n \frac{Z_i}{\pi(X_i)} (Y_i - \mu(Z_i, X_i))}{\sum_{i=1}^n \left( \frac{Z_i}{\pi(X_i)} \right)^2} \quad \hat{\phi}_0 = \frac{\sum_{i=1}^n \frac{1 - Z_i}{1 - \pi(X_i)} (Y_i - \mu(Z_i, X_i))}{\sum_{i=1}^n \left( \frac{1 - Z_i}{1 - \pi(X_i)} \right)^2}. \quad (5)$$

The corresponding estimators are derived from the fitted values from this model. For example

$$\begin{aligned} \hat{\mu}(1) &= \frac{1}{n} \sum_{i=1}^n \mu(1, X_i) + \hat{\phi}_1 \frac{1}{n} \sum_{i=1}^n \frac{1}{\pi(X_i)} \\ &= \frac{1}{n} \sum_{i=1}^n \mu(1, X_i) + \frac{\sum_{i=1}^n \frac{Z_i}{\pi(X_i)} (Y_i - \mu(Z_i, X_i))}{\sum_{i=1}^n \left( \frac{Z_i}{\pi(X_i)} \right)^2} \frac{1}{n} \sum_{i=1}^n \frac{1}{\pi(X_i)} \\ &= \frac{1}{n} \sum_{i=1}^n \mu(1, X_i) + \frac{\sum_{i=1}^n \frac{1}{\pi(X_i)}}{\sum_{i=1}^n \left( \frac{Z_i}{\pi(X_i)} \right)^2} \frac{1}{n} \sum_{i=1}^n \frac{Z_i}{\pi(X_i)} (Y_i - \mu(Z_i, X_i)) \end{aligned} \quad (6)$$

Now,

$$\frac{1}{n} \sum_{i=1}^n \frac{1}{\pi(X_i)} \xrightarrow{P} \mathbb{E} \left[ \frac{1}{\pi(X)} \right] \quad \frac{1}{n} \sum_{i=1}^n \left( \frac{Z_i}{\pi(X_i)} \right)^2 \xrightarrow{P} \mathbb{E} \left[ \left( \frac{Z}{\pi(X)} \right)^2 \right] = \mathbb{E} \left[ \frac{1}{\pi(X)} \right]$$

so the two estimators in (1) and (6) are asymptotically identical. The doubly robust estimating equation being solved here is the equivalent to (2), that is

$$\sum_{i=1}^n \left[ \hat{\mu}_A(1, X_i) + \frac{Z_i}{\pi(X_i)} (Y_i - \hat{\mu}_A(Z_i, X_i)) - \mu(1) \right] = 0 \quad (7)$$

where

$$\hat{\mu}_A(1, X_i) = \mu(1, X_i) + \hat{\phi}_1 \frac{1}{\pi(X_i)}.$$

By (3), we have that

$$\sum_{i=1}^n \frac{Z_i}{\pi(X_i)} (Y_i - \hat{\mu}_A(Z_i, X_i)) = \sum_{i=1}^n \left( Y_i - \mu(Z_i, X_i) - \hat{\phi}_1 \frac{Z_i}{\pi(X_i)} \right) \frac{Z_i}{\pi(X_i)} = 0.$$

The double robustness property relates to the correct specification of  $\mu_A(z, x)$  as the conditional expectation of  $Y$  given  $Z = z$  and  $X = x$  (or of  $\pi(x)$  as the propensity model). However, if in fact  $\mu(z, x)$  is the correct conditional expectation, then from (5) it is clear that  $\hat{\phi}_1 \xrightarrow{P} 0$  as  $n \rightarrow \infty$ .

**Simulation study:** In the following study we examine the performance of six estimators:

1.  $\mu(z, x)$  and  $\pi(x)$  correctly specified, augmented outcome regression estimator;
2.  $\mu(z, x)$  and  $\pi(x)$  correctly specified, direct AIPW estimator;
3.  $\pi(x)$  correctly specified,  $\mu(z, x)$  incorrectly specified, augmented outcome regression estimator;
4.  $\pi(x)$  correctly specified,  $\mu(z, x)$  incorrectly specified, direct AIPW estimator;
5.  $\mu(z, x)$  correctly specified,  $\pi(x)$  incorrectly specified, augmented outcome regression estimator;
6.  $\mu(z, x)$  correctly specified,  $\pi(x)$  incorrectly specified, direct AIPW estimator;

```

library(MASS)
set.seed(23987)
n<-1000000
D<-diag(c(0.25,0.5,0.75))
Mu<-c(1,-2,-1)
Sigma<-D %*% (matrix(c(1,0.9,-0.1,0.9,1,-0.2,-0.1,-0.2,1),3,3) %*% D)
X<-mvrnorm(n,mu=Mu,Sigma)
theta<-2.0
al<-c(6.0,-0.2,0.7,2)
Xal<-cbind(1,X[,1],X[,2],X[,1]*X[,2])
expit<-function(x){return(1/(1+exp(-x)))}
ps.true<-expit(Xal %*% al)
summary(ps.true)

+           V1
+   Min.      :0.08269
+  1st Qu.:0.54115
+   Median :0.64164
+    Mean  :0.63611
+   3rd Qu.:0.73598
+    Max.   :0.99964

Z<-rbinom(n,1,ps.true)
be<-c(0,theta,-2.0,1.2,0.6)
Xbe<-cbind(1,Z,X[,1],X[,2],X[,1]*Z)
mu.true<- Xbe %*% be
Y<-rnorm(n)*5+mu.true
X0<-cbind(1,0,X[,1],X[,2],X[,1]*0)
X1<-cbind(1,1,X[,1],X[,2],X[,1]*1)
mu0<-X0 %*% be
mu1<-X1 %*% be
ATE.true<-mean(mu1)-mean(mu0)
X1<-X[,1]; X2<-X[,2]; X3<-X[,3]

```

In this simulation, the true ATE value is 2.6.

```

Yoff<-Y-mu.true
C1<-Z/ps.true
phi1.hat<-sum(C1*Yoff)/sum(C1^2)
mu1.hat<-mean(mu1)+phi1.hat*mean(1/ps.true)
C0<-(1-Z)/(1-ps.true)
phi0.hat<-sum(C0*Yoff)/sum(C0^2)
mu0.hat<-mean(mu0)+phi0.hat*mean(1/(1-ps.true))
ATE.hat<-mu1.hat-mu0.hat

```

```

Yoff<-Y
C1<-Z/ps.true
phi1.hat<-sum(C1*Yoff)/sum(C1^2)
mu1.hat<-phi1.hat*mean(1/ps.true)

C0<-(1-Z)/(1-ps.true)
phi0.hat<-sum(C0*Yoff)/sum(C0^2)
mu0.hat<-phi0.hat*mean(1/(1-ps.true))
ATE.hat.mis<-mu1.hat-mu0.hat

c(ATE.hat,ATE.hat.mis)

+ [1] 2.616234 2.618082

```

The AOR and the AIPW estimators exhibit apparent consistent estimation; however, they are not numerically identical.

```

nreps<-10000
res.mat<-matrix(0,nrow=nreps,ncol=6)
nsamp<-10000

for(irep in 1:nreps){

  isub<-sample(1:n,size=nsamp,rep=T)

  Yoff<-Y[isub]-mu.true[isub]
  C1.sub<-Z[isub]/ps.true[isub]
  C0.sub<-(1-Z[isub])/(1-ps.true[isub])

  mu.sub<-mu.true[isub]
  Z.sub<-Z[isub]
  mu1.sub<-mu1[isub]
  mu0.sub<-mu0[isub]

  #AIPW estimator, correct specification
  ATE.direct<-mean(mu1.sub)-mean(mu0.sub)+mean(C1.sub*Yoff)-mean(C0.sub*Yoff)

  #AOR estimator, correct specification
  phi1.hat<-sum(C1.sub*Yoff)/sum(C1.sub^2)
  mu1.hat<-mean(mu1.sub)+phi1.hat*mean(1/ps.true[isub])
  phi0.hat<-sum(C0.sub*Yoff)/sum(C0.sub^2)
  mu0.hat<-mean(mu0.sub)+phi0.hat*mean(1/(1-ps.true[isub]))
  ATE.hat<-mu1.hat-mu0.hat

  #Incorrect mean model
  Yoff<-Y[isub]

  #AIPW estimator, incorrect mean specification
  ATE.direct.mis<-mean(C1.sub*Yoff)-mean(C0.sub*Yoff)

  #AOR estimator, incorrect mean specification
  phi1.hat<-sum(C1.sub*Yoff)/sum(C1.sub^2)
  mu1.hat<-phi1.hat*mean(1/ps.true[isub])
  phi0.hat<-sum(C0.sub*Yoff)/sum(C0.sub^2)
  mu0.hat<-phi0.hat*mean(1/(1-ps.true[isub]))
  ATE.hat.mis<-mu1.hat-mu0.hat

  #Incorrect PS model
  Yoff<-Y[isub]-mu.true[isub]

  ps.mis<-fitted(glm(Z[isub]~X1[isub],family=binomial))
  C1.sub<-Z[isub]/ps.mis
  C0.sub<-(1-Z[isub])/(1-ps.mis)

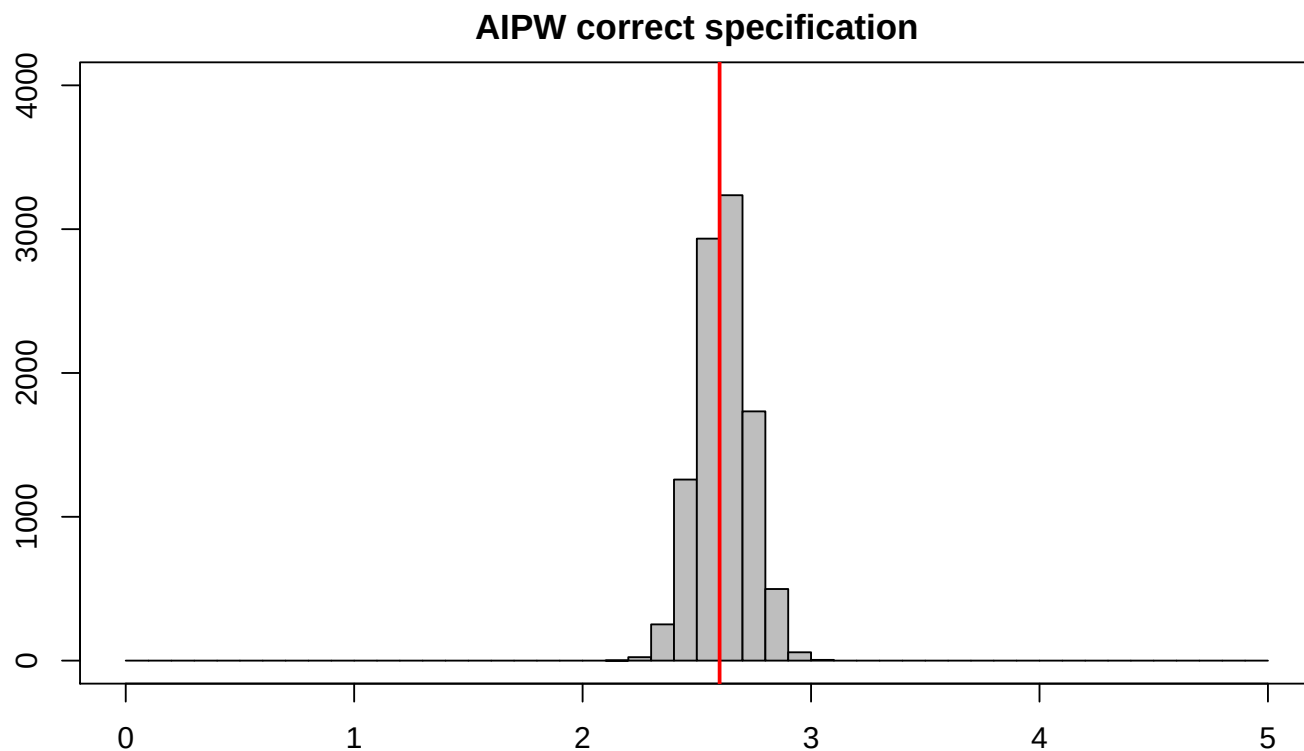
  #AIPW estimator incorrect ps specification
  ATE.direct.ps.mis<-mean(mu1.sub)-mean(mu0.sub)+mean(C1.sub*Yoff)-mean(C0.sub*Yoff)

  #AOR estimator, incorrect ps specification
  phi1.hat<-sum(C1.sub*Yoff)/sum(C1.sub^2)
  mu1.hat<-mean(mu1.sub)+phi1.hat*mean(1/ps.mis)
  phi0.hat<-sum(C0.sub*Yoff)/sum(C0.sub^2)
  mu0.hat<-mean(mu0.sub)+phi0.hat*mean(1/(1-ps.mis))
  ATE.ps.mis<-mu1.hat-mu0.hat

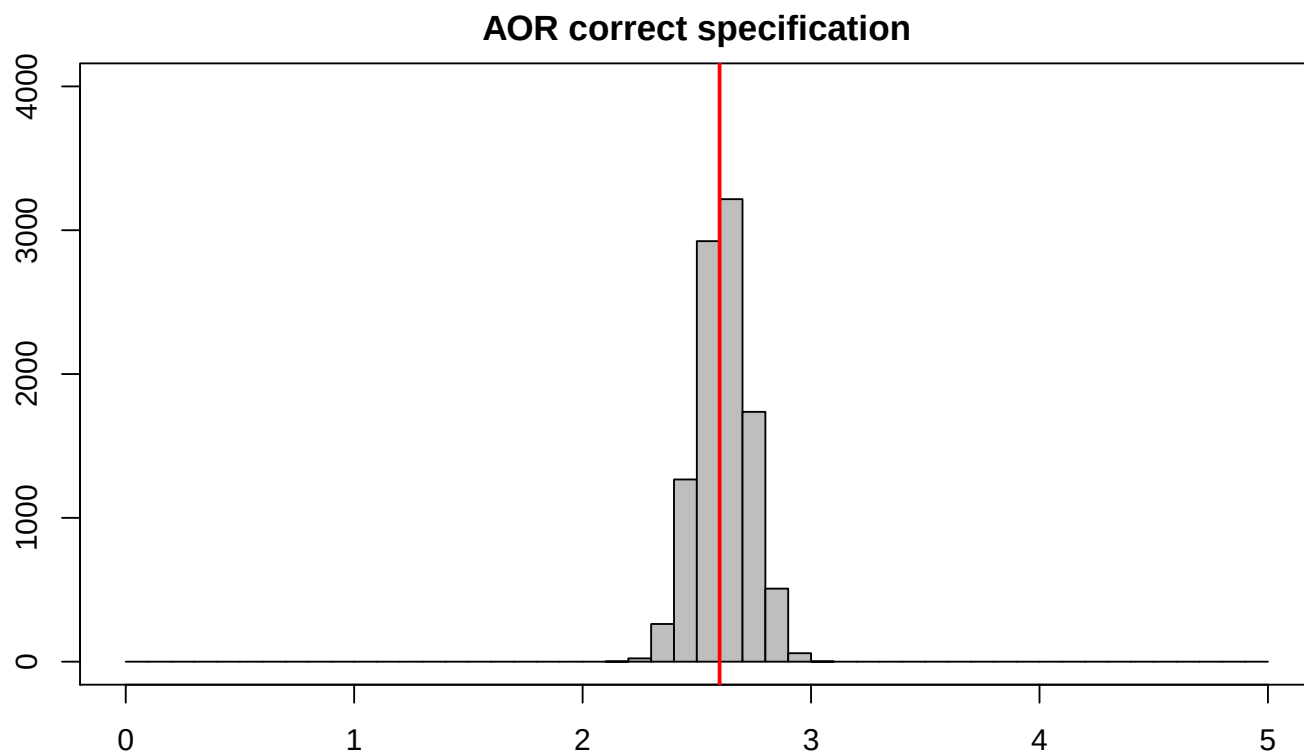
  res.mat[irep,]<-c(ATE.direct,ATE.hat,ATE.direct.mis,ATE.hat.mis,ATE.direct.ps.mis,ATE.ps.mis)
}

```

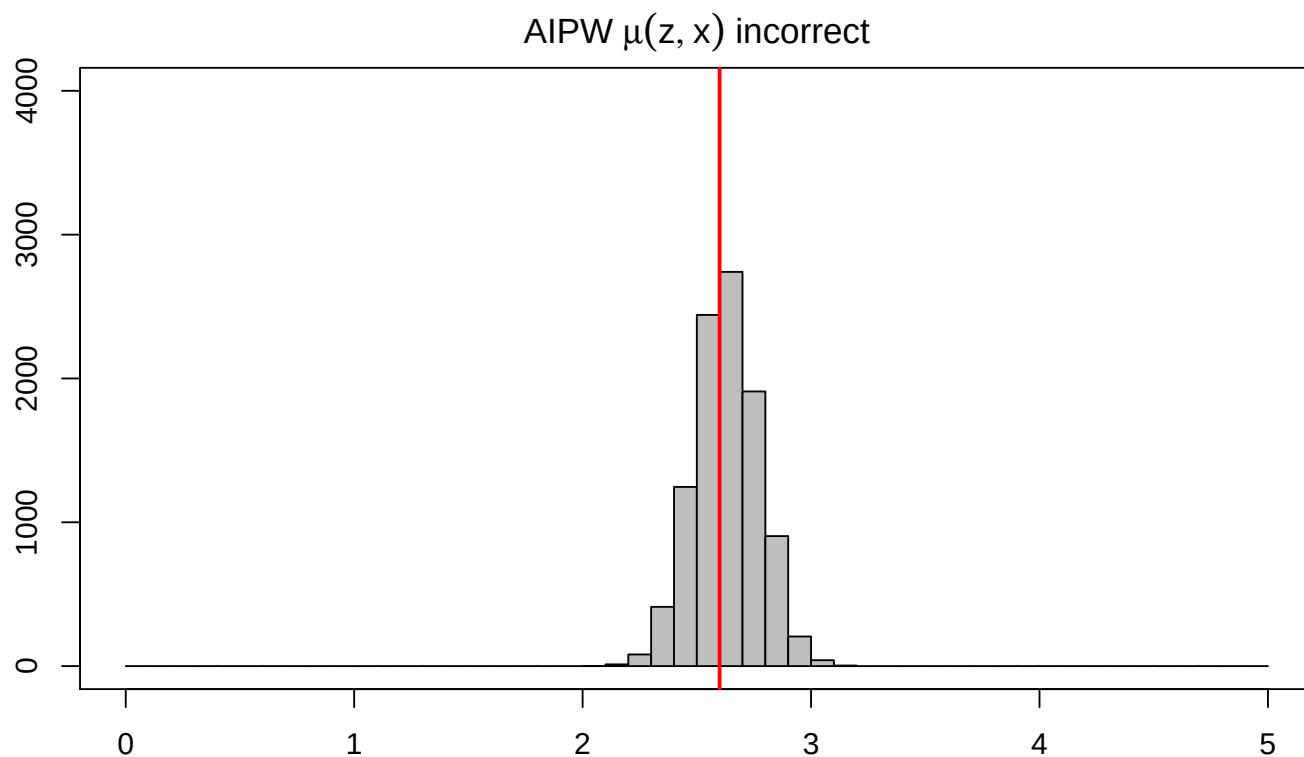
```
br<-seq(0.0,5.0,by=0.1);par(mar=c(2,2,2,0))
hist(res.mat[,1],ylim=range(0,4000),breaks=br,main='AIPW correct specification',col='gray',xlab='ATE')
box();abline(v=ATE.true,col='red',lwd=2)
```



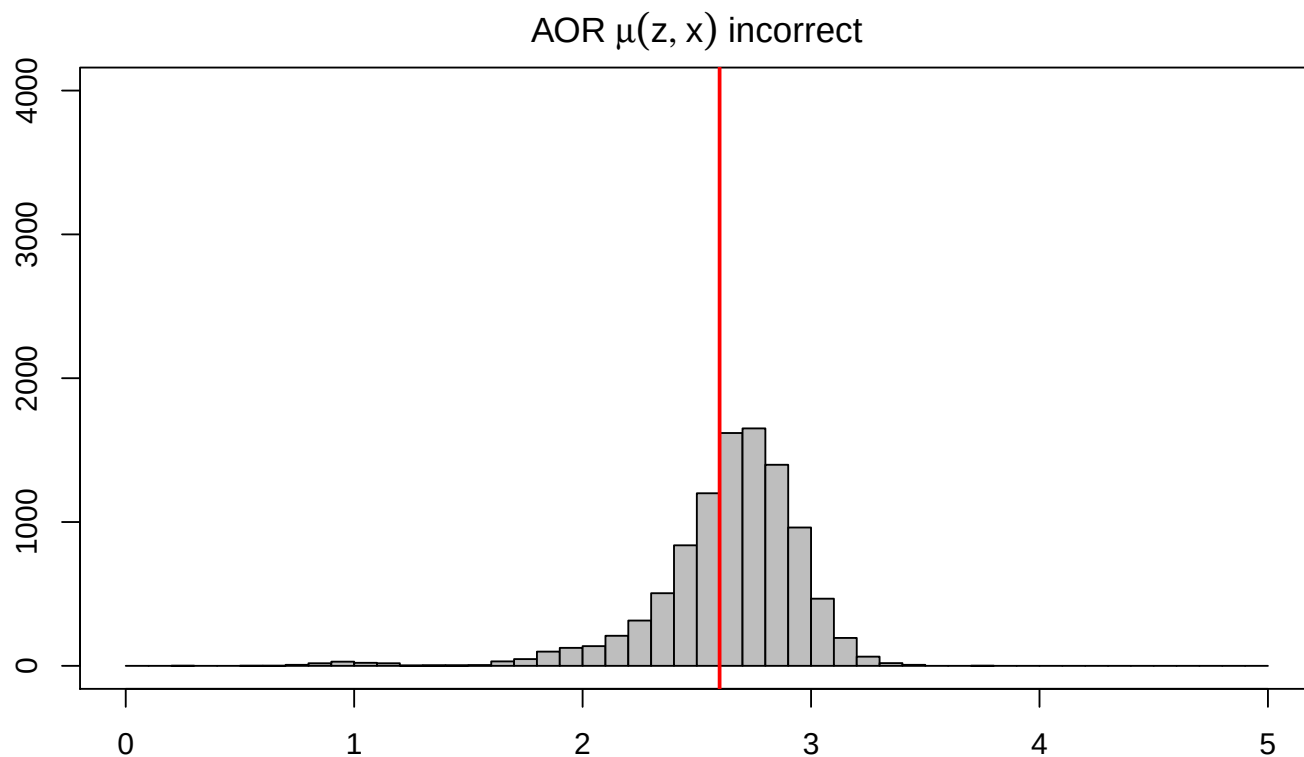
```
hist(res.mat[,2],ylim=range(0,4000),breaks=br,main='AOR correct specification',col='gray',xlab='ATE')
box();abline(v=ATE.true,col='red',lwd=2)
```



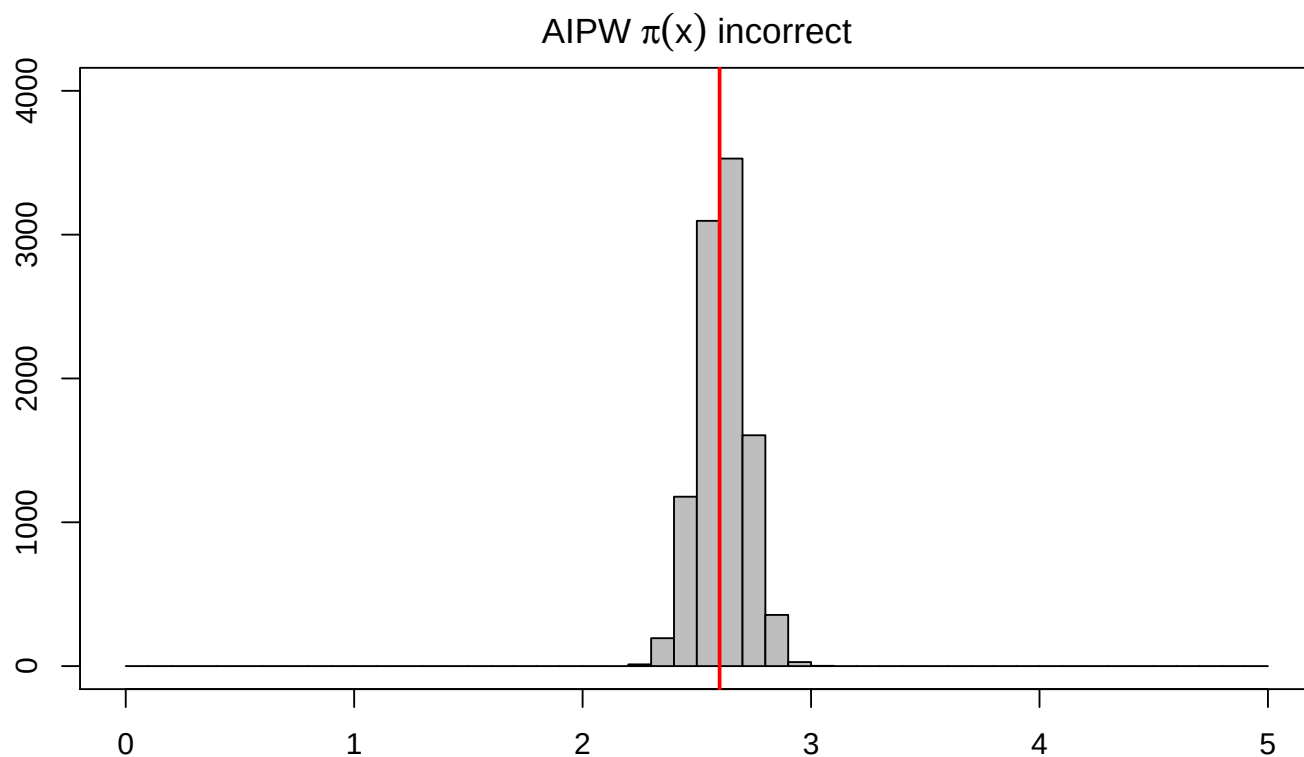
```
hist(res.mat[,3],ylim=range(0,4000),breaks=br,
     main=expression(paste('AIPW ',mu(z,x), ' incorrect')),col='gray',xlab='ATE')
box();abline(v=ATE.true,col='red',lwd=2)
```



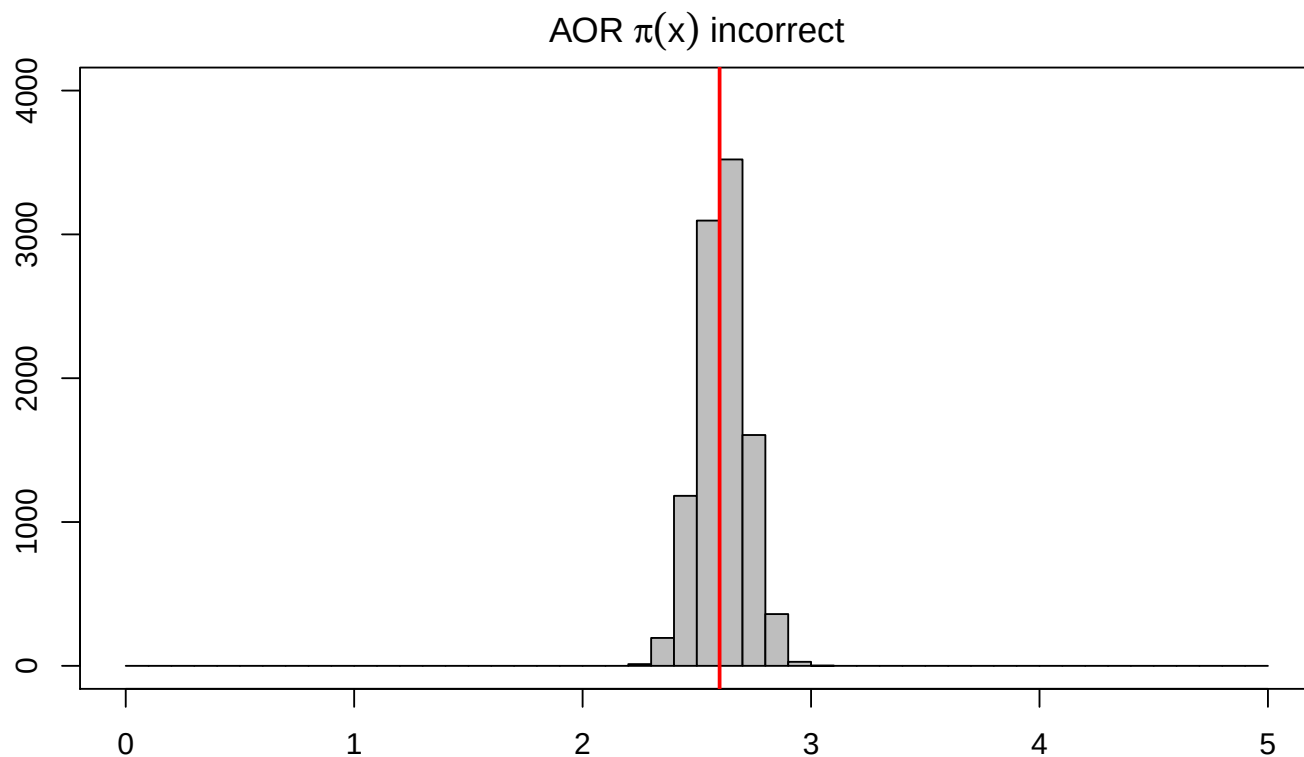
```
hist(res.mat[,4],ylim=range(0,4000),breaks=br,
     main=expression(paste('AOR ',mu(z,x), ' incorrect')),col='gray',xlab='ATE')
box();abline(v=ATE.true,col='red',lwd=2)
```



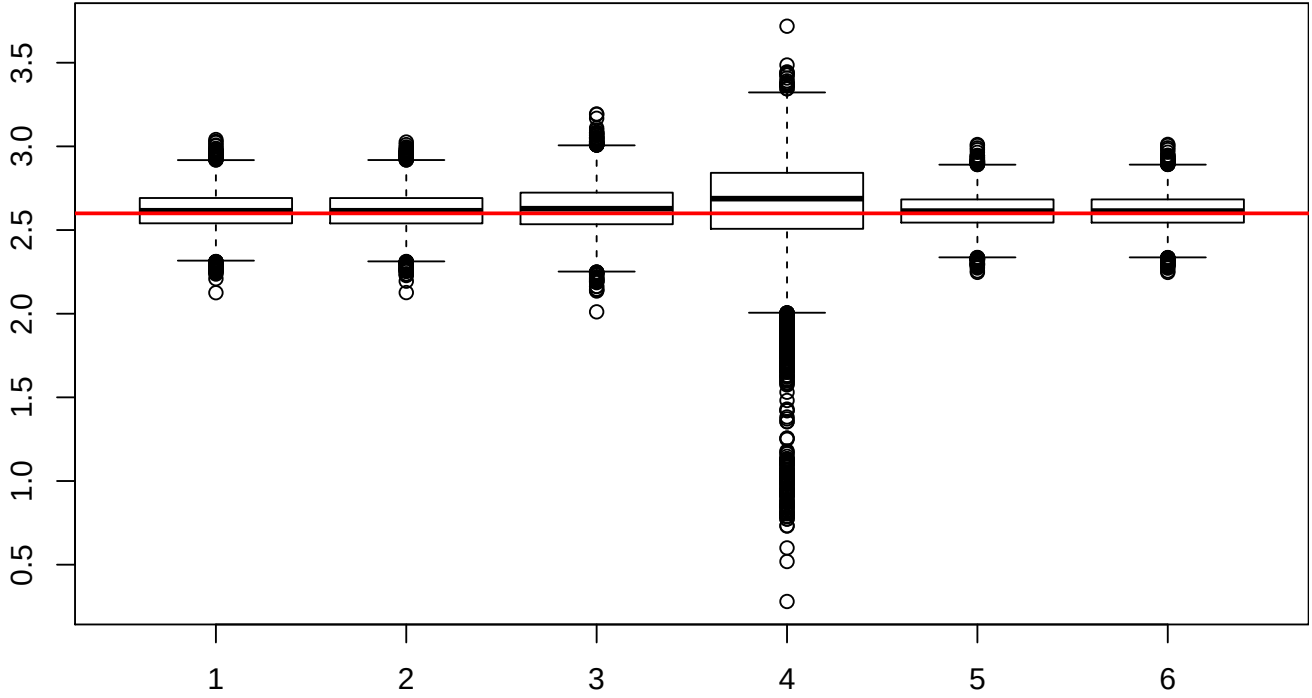
```
hist(res.mat[,5],ylim=range(0,4000),breaks=br,
      main=expression(paste('AIPW ',pi(x),' incorrect')),col='gray',xlab='ATE')
box();abline(v=ATE.true,col='red',lwd=2)
```



```
hist(res.mat[,6],ylim=range(0,4000),breaks=br,
      main=expression(paste('AOR ',pi(x),' incorrect')),col='gray',xlab='ATE')
box();abline(v=ATE.true,col='red',lwd=2)
```



```
boxplot(res.mat);abline(h=ATE.true,col='red',lwd=2)
```



```
apply(res.mat,2,mean)
+ [1] 2.616243 2.615996 2.629452 2.640492 2.613513 2.613525

apply(res.mat,2,var)*nsamp
+ [1] 129.5691 129.8919 196.2051 1048.8402 107.6084 107.9738

apply((res.mat-ATE.true)^2,2,mean)*nsamp
+ [1] 132.3003 132.5418 205.0507 1065.3934 109.5119 109.8806
```

The second Bang and Robins approach proposes taking  $\phi_0 = -\phi_1$ , and using the AOR model

$$\mathbb{E}[Y|Z, X] = \mu(Z, X) + \phi \left( \frac{Z}{\pi(X)} - \frac{1-Z}{1-\pi(X)} \right)$$

to produce the ATE estimator

$$\frac{1}{n} \sum_{i=1}^n \{ \mu(1, X_i) - \mu(0, X_i) \} + \hat{\phi} \sum_{i=1}^n \left( \frac{1}{\pi(X_i)} + \frac{1}{1-\pi(X_i)} \right).$$

By elementary calculations we have that

$$\hat{\phi} = \frac{\sum_{i=1}^n \left( \frac{Z_i}{\pi(X_i)} - \frac{1-Z_i}{1-\pi(X_i)} \right) (Y_i - \mu(Z_i, X_i))}{\sum_{i=1}^n \left( \frac{Z_i}{\pi(X_i)} - \frac{1-Z_i}{1-\pi(X_i)} \right)^2}$$

so that

$$\begin{aligned}\hat{\delta} &= \frac{1}{n} \sum_{i=1}^n \{\mu(1, X_i) - \mu(0, X_i)\} + \frac{1}{n} \sum_{i=1}^n \left( \frac{1}{\pi(X_i)} + \frac{1}{1 - \pi(X_i)} \right) \frac{\sum_{i=1}^n \left( \frac{Z_i}{\pi(X_i)} - \frac{1 - Z_i}{1 - \pi(X_i)} \right) (Y_i - \mu(Z_i, X_i))}{\sum_{i=1}^n \left( \frac{Z_i}{\pi(X_i)} - \frac{1 - Z_i}{1 - \pi(X_i)} \right)^2} \\ &= \frac{1}{n} \sum_{i=1}^n \{\mu(1, X_i) - \mu(0, X_i)\} + \frac{\sum_{i=1}^n \left( \frac{1}{\pi(X_i)} + \frac{1}{1 - \pi(X_i)} \right)}{\sum_{i=1}^n \left( \frac{Z_i}{\pi(X_i)} - \frac{1 - Z_i}{1 - \pi(X_i)} \right)^2} \frac{1}{n} \sum_{i=1}^n \left( \frac{Z_i}{\pi(X_i)} - \frac{1 - Z_i}{1 - \pi(X_i)} \right) (Y_i - \mu(Z_i, X_i)).\end{aligned}\tag{8}$$

As  $n \rightarrow \infty$  we have that

$$\frac{\sum_{i=1}^n \left( \frac{1}{\pi(X_i)} + \frac{1}{1 - \pi(X_i)} \right)}{\sum_{i=1}^n \left( \frac{Z_i}{\pi(X_i)} - \frac{1 - Z_i}{1 - \pi(X_i)} \right)^2} \xrightarrow{p} \frac{\mathbb{E} \left[ \frac{1}{\pi(X)} + \frac{1}{1 - \pi(X)} \right]}{\mathbb{E} \left[ \left( \frac{Z}{\pi(X)} - \frac{1 - Z}{1 - \pi(X)} \right)^2 \right]} = 1$$

so that  $\hat{\delta}$  is asymptotically identical to

$$\frac{1}{n} \sum_{i=1}^n \{\mu(1, X_i) - \mu(0, X_i)\} + \frac{1}{n} \sum_{i=1}^n \left( \frac{Z_i}{\pi(X_i)} - \frac{1 - Z_i}{1 - \pi(X_i)} \right) (Y_i - \mu(Z_i, X_i))$$

that is, the AIPW estimator.

```
nreps<-10000
res.mat2<-matrix(0,nrow=nreps,ncol=3)
phi.ests<-rep(0,nreps);Terms<-matrix(0,nrow=nreps,ncol=3)
nsamp<-10000

for(irep in 1:nreps){

  isub<-sample(1:n,size=nsamp,rep=T)

  Yoff<-Y[isub]-mu.true[isub]
  C.sub<-(Z[isub]/ps.true[isub]-(1-Z[isub])/(1-ps.true[isub]))

  mu.sub<-mu.true[isub]
  Z.sub<-Z[isub]
  mu1.sub<-mu1[isub]
  mu0.sub<-mu0[isub]

  #AOR estimator, correct specification
  phi.hat<-sum(C.sub*Yoff)/sum(C.sub^2)
  mu1.hat<-mean(mu1.sub)+phi.hat*mean(1/ps.true[isub])
  mu0.hat<-mean(mu0.sub)+phi.hat*mean(1/(1-ps.true[isub]))
  ATE.hat<-mu1.hat-mu0.hat

  phi.ests[irep,]<-phi.hat
  Terms[irep,1]<-mean(mu1.sub)-mean(mu0.sub)
  Terms[irep,2]<-sum(1/ps.true[isub]+1/(1-ps.true[isub]))/sum((Z.sub/ps.true[isub]-(1-Z.sub)/(1-ps.true[isub]))*Yoff)
  Terms[irep,3]<-mean((Z.sub/ps.true[isub]-(1-Z.sub)/(1-ps.true[isub]))*Yoff)

  #Incorrect mean model
  Yoff<-Y[isub]

  #AOR estimator, incorrect mean specification
```

```

phi.hat<-sum(C.sub*Yoff)/sum(C.sub^2)
mu1.hat<-phi.hat*mean(1/ps.true[isub])
mu0.hat<-phi.hat*mean(1/(1-ps.true[isub]))
ATE.hat.mis<-mu1.hat-mu0.hat

#Incorrect PS model
Yoff<-Y[isub]-mu.true[isub]
ps.mis<-fitted(glm(Z[isub]~X1[isub],family=binomial))

C.sub<-Z[isub]/ps.mis-(1-Z[isub])/(1-ps.mis)

#AOR estimator, incorrect ps specification
phi.hat<-sum(C.sub*Yoff)/sum(C.sub^2)
mu1.hat<-mean(mu1.sub)+phi.hat*mean(1/ps.mis)
mu0.hat<-mean(mu0.sub)-phi.hat*mean(1/(1-ps.mis))
ATE.ps.mis<-mu1.hat-mu0.hat

res.mat2[irep,]<-c(ATE.hat,ATE.hat.mis,ATE.ps.mis)
}

+ Warning: internal error -3 in R.decompress1
+ Error in eval(expr, envir, enclos): lazy-load database 'C:/Users/David/Dropbox/AIPW/Notes/cache/BR1.8e2f
is corrupt

```

```

br<-seq(1.5,3.5,by=0.05)
par(mar=c(3,2,2,0))
hist(res.mat2[,1],ylim=range(0,4000),breaks=br,
     main='AOR correct specification',col='gray',xlab='ATE')

+ Error in hist.default(res.mat2[, 1], ylim = range(0, 4000), breaks = br, : some 'x' not counted;
maybe 'breaks' do not span range of 'x'

box();abline(v=ATE.true,col='red',lwd=2)

+ Error in box(): plot.new has not been called yet
+ Error in int_abline(a = a, b = b, h = h, v = v, untf = untf, ...): plot.new has not been called
yet

br<-seq(1.5,3.5,by=0.05)
hist(res.mat2[,2],ylim=range(0,4000),breaks=br,
     main=expression(paste('AOR ',mu(z,x), ' incorrect')),col='gray',xlab='ATE')

+ Error in hist.default(res.mat2[, 2], ylim = range(0, 4000), breaks = br, : some 'x' not counted;
maybe 'breaks' do not span range of 'x'

box();abline(v=ATE.true,col='red',lwd=2)

+ Error in box(): plot.new has not been called yet
+ Error in int_abline(a = a, b = b, h = h, v = v, untf = untf, ...): plot.new has not been called
yet

br<-seq(1.5,3.5,by=0.05)
hist(res.mat2[,3],ylim=range(0,4000),breaks=br,
     main=expression(paste('AOR ',pi(x), ' incorrect')),col='gray',xlab='ATE')

+ Error in hist.default(res.mat2[, 3], ylim = range(0, 4000), breaks = br, : some 'x' not counted;
maybe 'breaks' do not span range of 'x'

box();abline(v=ATE.true,col='red',lwd=2)

+ Error in box(): plot.new has not been called yet
+ Error in int_abline(a = a, b = b, h = h, v = v, untf = untf, ...): plot.new has not been called
yet

```

```
boxplot(res.mat2);abline(h=ATE.true,col='red',lwd=2)

apply(res.mat2,2,mean)
+ [1] 0 0 0

apply(res.mat2,2,var)*nsamp
+ [1] 0 0 0

apply((res.mat2-ATE.true)^2,2,mean)*nsamp
+ [1] 67583.23 67583.23 67583.23

pairs(Terms)
```