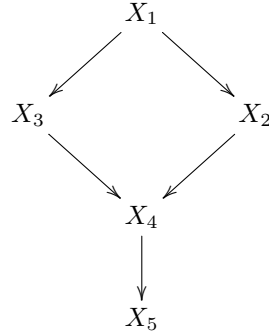


d-SEPARATION EXAMPLE

In the graph

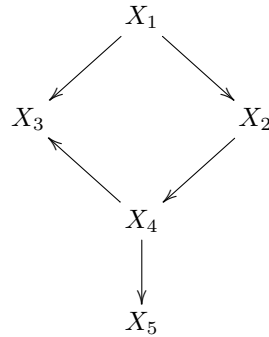


we have that there are two paths connecting X_2 and X_3 , namely

$$(X_2, X_1, X_3) \quad (X_2, X_4, X_3)$$

- the set $S \equiv \{X_1\}$ (conditionally) d-separates X_2 and X_3 as conditioning on X_1 blocks the path (X_2, X_1, X_3) ; we do not need to consider the second path here, as that is already blocked by the collider X_4 ;
- the set $S \equiv \{X_1, X_4\}$ does not (conditionally) d-separate X_2 and X_3 as conditioning on the collider X_4 opens the path;
- the set $S \equiv \{X_1, X_5\}$ does not (conditionally) d-separate X_2 and X_3 as conditioning on X_5 , a descendant of X_4 , opens the path at the collider.

However, in the graph



the same two paths are present, but

- the set $S \equiv \{X_1\}$ does not (conditionally) d-separate X_2 and X_3 as conditioning on X_1 blocks the path (X_2, X_1, X_3) ; but there is an open path through X_4 ;
- the set $S \equiv \{X_1, X_4\}$ (conditionally) d-separates X_2 and X_3 as conditioning on X_1 and X_4 blocks both paths;
- the set $S \equiv \{X_1, X_5\}$ does not (conditionally) d-separate X_2 and X_3 as the second path remains open at X_4 .

In general, when considering d-separation of two variables X and Y by a set of variables, S , we need to consider **all paths** between X and Y , not merely those involving S - however, in the first example X_4 is a collider so we do not need to consider the second path when taking $S \equiv \{X_1\}$.