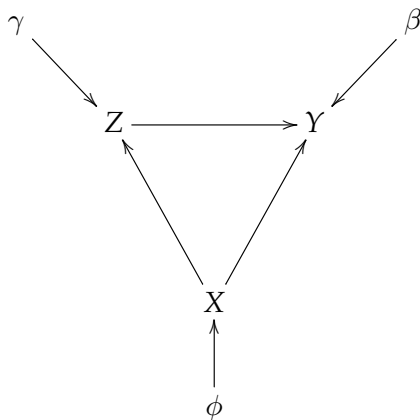


BAYESIAN CAUSAL INFERENCE – THE TRUTH

DAG with balancing B

The data generating Bayesian DAG is



DAG with balancing B (cont.)

Suppose

$$B = e(X, \gamma)$$

defines B as a deterministic function of X and γ . Then we have the distribution of B is degenerate conditional on (X, γ)

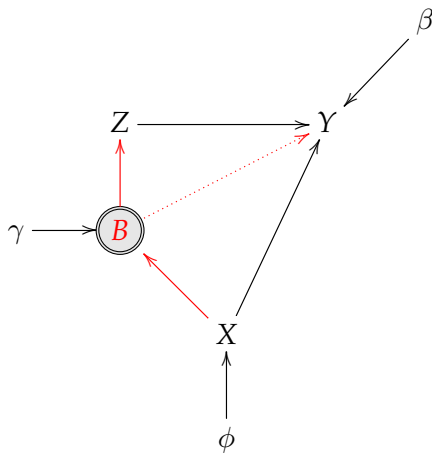
$$f_{B|X}(b|x, \gamma) = \begin{cases} 1 & b = e(x, \gamma) \\ 0 & \text{otherwise} \end{cases}.$$

Also

$$f_{Z|B}(z|b) = b^z(1-b)^{1-z} \quad z = 0, 1.$$

DAG with balancing B (cont.)

 deterministic node



DAG with balancing B (cont.)

Claim 1: γ and β are conditionally independent given X, Y, Z, B .

Proof: There are three paths connecting γ and β .

1. $\beta Y Z B \gamma$: on this path, Y is a collider, but conditioning on Z blocks the path.
2. $\beta Y B \gamma$: on this path, Y is a collider, but conditioning on B blocks the path. Conditioning on B requires knowledge of γ , as B is deterministically defined as $B = e(X, \gamma)$; however, once $B = b$ is fixed, knowledge of γ is no longer necessary.
3. $\beta Y X B \gamma$: on this path, Y is a collider, but conditioning on X blocks the path.

Thus all paths are blocked, and γ and β are conditionally independent.

Claim 2: γ is conditionally independent of Y given X, Z .

Proof: There are no open paths from γ to Y once Z and X are conditioned upon.