MATH 598/782 - PROJECT 1

Please submit your project by 6.00 pm (EDT) on Wednesday 23rd September by uploading a single pdf to myCourses.

You may use any computing language to perform the analyses. Please show your code in your solutions, or upload it as a separate file.

This question concerns Bayesian inference related to the Poisson distribution. As pointed out by Bernardo & Smith, the Poisson model with mass function

$$f_Y(y;\theta) = \frac{\theta^y \exp\{-\theta\}}{y!} \qquad y = 0, 1, 2, \ldots$$

and zero otherwise, for parameter $\theta > 0$, arises by considering observables taking values on the non-negative integers that yield certain summary statistics, or as the limiting case of a discrete selection (multinomial) model.

(a) Derive the form of the Bayesian posterior distribution for a sample $y_1, \ldots, y_n$ of size n for the prior models

(i) $\pi_0(\theta) \equiv Gamma(\alpha, \beta)$ for $\alpha, \beta > 0$;

(ii) $\pi_0(\theta)$ determined by the assumption that $\phi = \log\theta$ is $Normal(\eta, \tau^2)$ distributed *a priori*.

6 MARKS

(b) For values of the hyperparameters $\alpha, \beta, \eta, \tau^2$ of your choosing, plot the posterior density $\pi_n(\theta)$ under the two priors in part (a) for the following data, a sample of size $n = 50$, that are displayed in aggregate form

y	0	1	2	3	4	5	6
Count	2	6	7	16	11	6	2

That is, there were two observations with $y = 0$, six with $y = 1$ and so on. 4 MARKS