

MATH 598 - PROJECT 1

Please submit your project by 11.59 pm on Wednesday 13th February by uploading a single pdf to myCourses.

This question concerns Bayesian inference related to the Exponential distribution. As pointed out by Bernardo & Smith, the Exponential model with density

$$f_Y(y; \theta) = \theta \exp\{-\theta y\} \quad y > 0$$

and zero otherwise, for parameter $\theta > 0$, arises by considering observables taking values on $\mathbb{R}^+$ that exhibit origin invariance.

(a) Find the form of the Bayesian posterior distribution for a sample $y_1, \dots, y_n$ of size n for the following prior models:

- (i) $\pi_0(\theta) \equiv \text{Gamma}(\alpha, \beta)$ for $\alpha, \beta > 0$;
- (ii) $\pi_0(\theta) \equiv \text{Uniform}(0, a)$ for $a > 0$;
- (iii) $\pi_0(\theta)$ takes the mixture form

$$\pi_0(\theta) = \frac{1}{2} \exp\{-\theta\} + \exp\{-2\theta\} \quad \theta > 0$$

and zero otherwise.

and, for values of the hyperparameters α, β, a of your choosing, plot the posterior density $\pi_n(\theta)$ for the following data

3.33 2.04 1.96 4.72 1.13 0.12 4.06 0.13 1.79 0.11

the realized values of $n = 10$ exchangeable random variables $Y_1, \dots, Y_{10}$.

15 MARKS

- (b) Compute and plot the posterior density for parameter $\mu = 1/\theta$ for case (a) (i), for the data provided and with your chosen hyperparameters. 5 MARKS
- (c) Compute and plot the joint predictive density for Y_{11} and Y_{12} based on the observed data for case (a) (i), with the data provided and your chosen hyperparameters. 4 MARKS
- (d) A $(1 - \gamma) \times 100\%$ credible interval for θ is an interval in the computed posterior distribution that contains probability $1 - \gamma$. Compute a 95% credible interval for each of the cases in (a), for the data provided and your chosen hyperparameters. 6 MARKS

In your solutions, show the code used to produce the plots and numerical results.