MATH 559 - EXERCISES 2

Not for Assessment

1. Suppose $Y_1, \ldots, Y_n$ are realizations from an exchangeable binary sequence. Using the Jeffreys prior for parameter
$$\theta = \mathbb{E}_Y[Y]$$
find an approximation to the posterior distribution $\pi_n(\theta)$ for large n.

2. Suppose that in a Bayesian model, we have that
$$f_Y(y;\theta) = \mathbb{1}_{(\theta,\infty)}(y)\exp\{-(y-\theta)\} \quad y \in \mathbb{R}$$
for $\theta \in \Theta \equiv \mathbb{R}^+$. Suppose that the prior is $\pi_0(\theta) \equiv Exponential(2)$. Find the posterior, $\pi_n(\theta)$, based on a sample $y_1, \ldots, y_n$.

3. Suppose that in a Bayesian model, we have that
$$f_Y(y;\theta) = \mathbb{1}_{(0,\infty)}(y)\frac{y}{\theta^2}\exp\left\{-\frac{y^2}{2\theta^2}\right\} \quad y \in \mathbb{R}$$
for $\theta \in \Theta \equiv \mathbb{R}^+$. Using a prior of your choosing, find the posterior, $\pi_n(\theta)$ based on a sample $y_1, \ldots, y_n$.

4. Suppose exchangeable sequences $\{Y_{1n}, Y_{2n}\}$ are such that given parameters $\theta_1, \theta_2, \sigma^2$
$$Y_{ji} \sim Normal(\theta_j, \sigma^2) \qquad j = 1, 2,\ i = 1, \ldots, n_j$$
are independent. Suppose that a proper, conjugate prior specification with
$$\pi_0(\theta_1, \theta_2, \sigma^2) = \pi_0(\sigma^2)\pi_0(\theta_1|\sigma^2)\pi_0(\theta_2|\sigma^2)$$
is used. Compute the posterior distribution for
$$\psi = \theta_2 - \theta_1.$$

5. Suppose exchangeable sequences $\{\mathbf{Y}_n\}$ are assumed to arise from a Bayesian model with
$$f_{\mathbf{Y}}(\mathbf{y};\boldsymbol{\theta}) \equiv Normal_2(\boldsymbol{\theta}, \Sigma_0)$$
where $\mathbf{Y}_1, \ldots \mathbf{Y}_n$ are 2×1 random vectors that are conditionally independent given parameters $\boldsymbol{\theta} = (\theta_1, \theta_2)^\top$, where Σ_0 is a known covariance matrix.

   (i) Find the posterior distribution for $\boldsymbol{\theta}$ if a conjugate prior is used.
   (ii) Find the marginal posteriors for θ_1 and for θ_2.
   (iii) Find the conditional posterior for θ_2 given θ_1.

6. Show that, in general, Bayes estimators defined by expected loss minimization are not invariant to 1-1 transformations; that is, if $\widehat{\theta}_{nB}$ is a Bayes estimator of θ, and $\phi = g(\theta)$ is 1-1 reparameterization of the model, then
$$\widehat{\phi}_{nB} \neq g(\widehat{\theta}_{nB})$$
in general.