# MATH 559 - ASSIGNMENT 2

***Please submit your solutions by 11.59 pm (EDT) on Friday 20th October by uploading a single pdf to myCourses.***

A total of $n = 24$ times are recorded, each measuring the operating life (time from installation to failure) of a different electrical component. Suppose that the times are presumed to be realizations from an infinitely exchangeable sequence $\{Y_i\}$ such that in the de Finetti representation.

$$f_Y(y;\theta) = \theta e^{-\theta y} \qquad y > 0$$

where $\theta \in \Theta = \mathbb{R}^+ \equiv \{t \in \mathbb{R} : t > 0\}$; that is, given $\theta$, $Y_1, \ldots, Y_n$ are presumed independent and identically distributed, with $Y_i \sim Exponential(\theta)$. It is recorded that

$$s_n \equiv \sum_{i=1}^{n} y_i = 21.17.$$

(a) Find the posterior distribution for $\theta$ if the prior is chosen to be $Gamma(2, 0.1)$. 3 MARKS

(b) Find the Jeffreys prior for $\theta$, and find the posterior under this prior. 5 MARKS

(c) For a quadratic loss $L(t, \theta) = (t - \theta)^2$, compute the Bayesian estimates (ie the numerical values) from the posteriors in (a) and (b). 2 MARKS

(d) For a quadratic loss $L(t, \theta) = (t - \theta)^2$, plot (using any suitable software) the minimum expected posterior loss

$$\min_t \int_0^\infty L(t,\theta)\pi_n(\theta)\, d\theta$$

if the posterior is computed under the prior $\pi_0(\theta) \equiv Gamma(2, \beta_0)$ for $0 < \beta_0 \leq 100$. 4 MARKS

(e) Find (using any suitable software) the *one-sided* Bayesian credible interval $\mathcal{C}_{0.95} \equiv (0, c)$ such that

$$\int_0^c \pi_n(\theta)\, d\theta = 0.95.$$

for the posterior in (a). 4 MARKS

(f) It is also recorded that

$$t_n \equiv \sum_{i=1}^{n} y_i^2 = 30.37.$$

Comment on the plausibility of the presumed Exponential model. 2 MARKS