MATH 559 – FINAL PROJECT

Due date: 11.59 pm, Friday 4th December

Please upload a single pdf to myCourses.

1. The continuous univariate pdf

$$f(x) = c\log(x+1)\exp\{-x^2/2\} \qquad x > 0$$

for constant $c > 0$ is a non-standard form. Here $\log x \equiv \ln x$.

Using Monte Carlo methods based on

(a) Rejection sampling — 10 MARKS

(b) The Metropolis-Hastings Algorithm — 10 MARKS

construct estimates of $\mathbb{E}_f[X]$; in each case, compare the variances of your Monte Carlo estimators over replicated estimation runs.

2. A Bayesian analysis based on the non-linear regression model

$$Y_i = \mu(x_i;\beta) + \epsilon_i$$

is required, where $\beta = (\beta_0, \beta_1, \beta_2, \beta_3)^\top$ and

$$\mu(x;\beta) = \beta_1 + (\beta_2 - \beta_1)\frac{\exp\{\beta_3(x-\beta_0)\}}{1+\exp\{\beta_3(x-\beta_0)\}}$$

with $\beta_0 \in \mathbb{R}$ and $\beta_1, \beta_2, \beta_3 > 0$, and $\epsilon_i \sim Normal(0, \sigma^2)$ are independent residual errors.

Carry out a Bayesian analysis of the data found in either

- the `.csv` file stored on the clickable link NLRC
- the `.txt` file stored on the clickable link NLRT

Specifically, give details of

(a) the form of the posterior distribution based on the prior that you choose; — 4 MARKS

(b) the analysis approach taken; — 6 MARKS

(c) the computational implementation and diagnostic checks carried out; — 6 MARKS

(d) the statistical conclusions made. — 4 MARKS

You may use modified material from lectures and knitr sheets, or AI tools, to construct your code and solutions, but you must give detailed explanation of your approach and specific algorithmic choices you make. You should include annotated code and results where appropriate.