MATH 557 - ASSIGNMENT 1

To be handed in not later than 10pm, Sunday 29th January 2017.
Please submit your solutions in pdf via myCourses.

Minimal sufficiency: A sufficient statistic T is *minimally sufficient* for parameter θ in model $F_X(x;\theta)$ (or $f_X(x;\theta)$) if it is a function of every other sufficient statistic. Further, T is minimally sufficient if, for two realizations $\mathbf{x} = x_{1:n}$ and $\mathbf{y} = y_{1:n}$,

$$T(\mathbf{x}) = T(\mathbf{y}) \qquad \Longleftrightarrow \qquad \frac{f_X(\mathbf{x};\theta)}{f_X(\mathbf{y};\theta)} \text{ does not depend on } \theta.$$

1. Consider discrete rvs $X_1, \ldots, X_n$ iid from the probability distribution with mass function

$$f_X(x;\theta) = \frac{\theta^x}{x! \sum_{z=0}^{\theta} \frac{\theta^z}{z!}} \qquad x = 0, 1, \ldots, \theta$$

and zero otherwise, for $\theta \in \mathbb{N} = \{1, 2, \ldots\}$.

Find a minimal sufficient statistic for θ based on $X_1, \ldots, X_n$, demonstrating minimal sufficiency. 4 MARKS

2. Suppose that $X_1, \ldots, X_n$ are iid from a $Uniform(-\lambda, \lambda)$ distribution. Find a sufficient statistic for λ, and state, giving justification, whether it is minimally sufficient. 4 MARKS

3. A fair coin is tossed, and the result is recorded as random variable Z.

   - If a HEAD is observed, a random sample $X_1, \ldots, X_n \sim Bernoulli(\theta)$ is collected, with the sample size n fixed.
   - If a TAIL is observed, a random sample $X_1, X_2, \ldots \sim Bernoulli(\theta)$ is collected until k successes (that is, where $X_i = 1$) are obtained, for some $k < n$.

   Let N denote the random variable recording the total number of Xs observed, and let M denote the number of successes in the observed Xs.

   (a) Find a sufficient statistic for θ. 4 MARKS

   (b) Show that the conditional distribution of Z, given N and M, does not depend on θ. 4 MARKS

4. Suppose that $Y_1, \ldots, Y_n$ are independent random variables with

$$Y_i \sim Binomial(m_i, \psi_i) \qquad \psi_i = \frac{\exp\{\beta_0 + \beta_1 x_i\}}{1 + \exp\{\beta_0 + \beta_1 x_i\}}$$

for some fixed real constants $x_1, \ldots, x_n$, and positive integers $m_1, \ldots, m_n$. Here, the parameter space is $\mathbb{R}^2$.

Find a minimal sufficient statistic for $\boldsymbol{\theta} = (\beta_0, \beta_1)^\top$, demonstrating minimal sufficiency.

4 MARKS