

MATH 557 - ASSIGNMENT 4

To be handed in not later than 5pm, 8th April 2010.

Please hand in during lecture, to Burnside 1225, or to the Mathematics Office Burnside 1005

1 Suppose that $X_1, \dots, X_n \sim \text{Poisson}(\lambda)$ are a random sample.

(a) Find a UMP test at level $\alpha = 0.05$ of the hypotheses

$$H_0 : \lambda = 1$$

$$H_1 : \lambda = 2$$

2 MARKS

(b) Find a UMP test at level $\alpha = 0.05$ of the hypotheses

$$H_0 : \lambda \leq 0.5$$

$$H_1 : \lambda > 0.5$$

4 MARKS

(c) Report the result of the test in (a) for data set of size $n = 10$.

2 1 2 3 2 1 0 2 3 2

2 MARKS

(d) Construct a test of the hypotheses

$$H_0 : \lambda = 3$$

$$H_1 : \lambda \neq 3$$

at level $\alpha = 0.01$. Is this test UMP? Justify your answer.

2 MARKS

2 Suppose that $X_1, \dots, X_n$ is a random sample from density

$$f_{X|\theta}(x|\gamma, \theta) = \frac{\gamma}{\theta} x^{\gamma-1} \exp\{-x^\gamma/\theta\} \mathbb{1}_{(0,\infty)}(x)$$

where $\gamma, \theta > 0$.

(a) Suppose that $\theta = 1$ is known. Find an interval estimator for γ with confidence level $1 - \alpha$.

3 MARKS

(b) Suppose that both γ and θ are unknown. Find a $1 - \alpha$ confidence set for (γ, θ) , $C(\underline{X})$, using the random variable $T(\underline{X}, \gamma, \theta)$ defined by

$$T(\underline{X}, \gamma, \theta) = \prod_{i=1}^n (X_i^\gamma / \theta)$$

where

$$C(\underline{X}) = \{(\gamma, \theta) : c_1 \leq T(\underline{X}, \gamma, \theta) \leq c_2\}$$

is a set having (frequentist) probability $1 - \alpha$.

3 MARKS

3 Suppose that $X_1, \dots, X_n \sim \text{Beta}(\theta, 1)$ are a random sample. Find the shortest $1 - \alpha$ confidence interval for θ based on a θ -sufficient statistic.

4 MARKS