

MATH 557 - ASSIGNMENT 1

To be handed in not later than 5pm, 26th January 2009.

Please hand in during lectures, to Burnside 1225, or to the Mathematics Office Burnside 1005

1 Consider a random sample $X_1, \dots, X_n$ from the probability distribution with mass function

$$f_{X|\theta}(x|\theta) = \frac{\theta^x}{x! \sum_{y=0}^{\theta} \frac{\theta^y}{y!}} \quad x = 0, 1, \dots, \theta$$

and zero otherwise, for $\theta \in \mathbb{N} = \{1, 2, \dots\}$.

Find a minimal sufficient statistic for θ , demonstrating minimal sufficiency.

5 MARKS

2 A fair coin is tossed, and the result is recorded as random variable Z .

- If a HEAD is observed, a random sample $X_1, \dots, X_n \sim \text{Bernoulli}(\theta)$ is collected, with the sample size n fixed.
- If a TAIL is observed, a random sample $X_1, X_2, \dots \sim \text{Bernoulli}(\theta)$ is collected until k successes (that is, where $X_i = 1$) are obtained, for some $k < n$.

Let N denote the random variable recording the total number of X s observed, and let M denote the number of successes in the observed X s.

(a) Find a sufficient statistic for θ .

4 MARKS

(b) Show that the conditional distribution of Z , given N and M , does not depend on θ .

4 MARKS

3 Suppose that $Y_1, \dots, Y_n$ are independent random variables with

$$Y_i \sim \text{Binomial}(m_i, \psi_i)$$

where

$$\psi_i = \frac{\exp\{\beta_0 + \beta_1 x_i\}}{1 + \exp\{\beta_0 + \beta_1 x_i\}}$$

for some fixed real constants $x_1, \dots, x_n$, and positive integers $m_1, \dots, m_n$. Here, the parameter space is $\Theta = \mathbb{R}^2$.

(a) Find a minimal sufficient statistic for $\underline{\theta} = (\beta_0, \beta_1)$, demonstrating minimal sufficiency.

5 MARKS

(b) Suppose that the constants $x_1, \dots, x_n$ take the value 0 or 1. In terms of the distributions of the $Y_1, \dots, Y_n$, give an interpretation of the parameter β_1 .

2 MARKS