556: MATHEMATICAL STATISTICS I
SUMMARY OF KEY DEFINITIONS, METHODS AND RESULTS

1. **Probability and Random Variables**

   - Definitions and results of basic probability theory
   - Definition and properties of univariate pmf/pdf/cdfs
   - *Key Techniques: Normalization, relationships between f_X and F_X.*

2. **Univariate and Multivariate Distributions**

   - List of standard distributions and the links between them
   - Definition and properties of multivariate pmf/pdf/cdfs
   - Computation of probabilities $\Pr[\underset{\sim}{X} \in \mathcal{A}]$
   - Independence
   - Conditional distributions
   - Examples of multivariate distributions
   - *Key Techniques: Normalization, Marginalization, Conditioning*

3. **Transformations and Expectations**

   - Univariate and multivariate transformations
   - Expectations and their properties
   - Moments
   - Generating functions
     - moment generating functions
     - cumulant generating functions
     - factorial moment/probability generating functions
     - characteristic functions
   - Characteristic functions
     - properties and uses
     - inversion formulae
     - diagnosing discrete/continuous distributions
   - Covariance and correlation
   - Law of iterated expectations
   - *Key Techniques: Univariate and Multivariate Transformations, Univariate and Multivariate Expectations, Use of Generating Functions to compute Moments and Characterize Distributions, Computation of Covariance and Correlation*

4. **Families of Distributions**

   - Location/Scale Family
   - Exponential Family
     - Form and terminology
     - Expectation and variance results
     - Tilting construction
   - Convolution Family
   - Hierarchical Models

5. **Inequalities**

    - Concentration inequalities
        - Chebychev lemma
        - Markov inequality
        - Chebychev inequality
        - Chernoff Bounds
    - Inequalities for multiple random variables
        - Hölder's inequality
        - Cauchy-Schwarz inequality
        - Minkowski's inequality
    - Jensen's inequality
    - *Key Techniques: Formal proofs of the technical results*

6. **Sampling Distributions**

    - Distributions of expectation and variance random variables
    - Results for specific families (location/scale, Exponential Family)
    - Results for the Normal family
    - Order statistics and sample quantiles
    - *Key Techniques: Proofs for the different families, proofs for and applications of results for extreme order statistics (min and max)*

7. **Stochastic Convergence**

    - Convergence in distribution
    - Convergence in probability
        - Weak Law of Large Numbers
    - The Central Limit Theorem
    - The Delta Method
    - *Key Techniques: Applications of definitions, use of CLT and Delta method to construct asymptotic or large sample distributions*

Please note that this list is not exhaustive.