MATH 556 - ASSIGNMENT 3

To be handed in not later than 11.59pm, 15th November 2019.
Please submit your solutions as pdf via myCourses.

1. Suppose that X and Y are positive, independent continuous random variables with cdfs F_X and F_Y. Show that

$$P[X < Y] = \int_0^1 F_X\left(F_Y^{-1}(t)\right) dt.$$

where F_Y^{-1} is the inverse function for the 1-1 function F_Y.

Hint: Sketch the region in the positive quadrant corresponding to the required probability. Recall that

$$F_X(x) = \int_0^x f_X(t)\, dt.$$

6 Marks

2. Suppose that Z_1 and Z_2 are independent random variables each having an $Exponential(1)$ distribution. Find the joint pdf of random variables Y_1 and Y_2 defined by

$$Y_1 = \frac{Z_1}{Z_1 + Z_2} \qquad Y_2 = Z_1 + Z_2.$$

5 Marks

Are Y_1 and Y_2 independent ? Justify your answer. 1 Mark

3. Consider the distribution for continuous random variable X with pdf specified via the two dimensional parameter $\theta = (\psi, \gamma)$ as

$$f_X(x; \psi, \gamma) = \mathbb{1}_{(0,\infty)}(x)\sqrt{\frac{1}{2\pi\gamma x^3}} \exp\left\{-\frac{1}{2}\psi^2\gamma x + \psi - \frac{1}{2\gamma x}\right\}$$

for $\psi, \gamma > 0$ and

(a) Is this a location-scale family distribution ? Justify your answer. 2 Marks

(b) Is this an Exponential Family distribution ? Justify your answer. 2 Marks

(c) For this model, the result concerning the expected score holds, that is

$$\mathbb{E}_X[\mathbf{S}(X;\theta)] = \mathbf{0} \qquad (2 \times 1)$$

where

$$\mathbf{S}(x;\theta) = \begin{pmatrix} S_1(x;\theta) \\ S_2(x;\theta) \end{pmatrix} = \begin{pmatrix} \dfrac{\partial}{\partial\psi} \log\{f_X(x;\psi,\gamma)\} \\ \dfrac{\partial}{\partial\gamma} \log\{f_X(x;\psi,\gamma)\} \end{pmatrix}$$

Using this result, find $\mathbb{E}_X[X]$ and $\mathbb{E}_X[1/X]$ 4 Marks