556: MATHEMATICAL STATISTICS I SUMMARY OF KEY DEFINITIONS, METHODS AND RESULTS

1. Probability and Random Variables

- Definitions and results of basic probability theory
- Definition and properties of univariate pmf/pdf/cdfs
- Key Techniques: Normalization, relationships between f_X and F_X .

2. Univariate and Multivariate Distributions

- List of standard distributions and the links between them
- Definition and properties of multivariate pmf/pdf/cdfs
- Computation of probabilities $\Pr[X \in \mathcal{A}]$
- Independence
- Conditional distributions
- Examples of multivariate distributions
- Key Techniques: Normalization, Marginalization, Conditioning

3. Transformations and Expectations

- Univariate and multivariate transformations
- Expectations and their properties
- Moments
- Generating functions
 - moment generating functions
 - cumulant generating functions
 - factorial moment/probability generating functions
 - characteristic functions
- Characteristic functions
 - properties and uses
 - inversion formulae
 - diagnosing discrete/continuous distributions
- Covariance and correlation
- Law of iterated expectations
- Key Techniques: Univariate and Multivariate Transformations, Univariate and Multivariate Expectations, Use of Generating Functions to compute Moments and Characterize Distributions, Computation of Covariance and Correlation

4. Families of Distributions

- Location/Scale Family
- Exponential Family
 - Form and terminology
 - Expectation and variance results
 - Tilting construction
- Convolution Family
- Hierarchical Models

5. Inequalities

- Concentration inequalities
 - Chebychev lemma
 - Markov inequality
 - Chebychev inequality
 - Chernoff Bounds
- Inequalities for multiple random variables
 - Hölder's inequality
 - Cauchy-Schwarz inequality
 - Minkowski's inequality
- Jensen's inequality
- Key Techniques: Formal proofs of the technical results

6. Sampling Distributions

- Distributions of expectation and variance random variables
- Results for specific families (location/scale, Exponential Family)
- Results for the Normal family
- Order statistics and sample quantiles
- Key Techniques: Proofs for the different families, proofs for and applications of results for extreme order statistics (min and max)

7. Stochastic Convergence

- Convergence in distribution
- Convergence in probability
 - Weak Law of Large Numbers
- The Central Limit Theorem
- The Delta Method
- *Key Techniques: Applications of definitions, use of CLT and Delta method to construct asymptotic or large sample distributions*

Please note that this list is not exhaustive.