MATH 556 - ASSIGNMENT 3: SOLUTIONS ADDENDUM

Addendum: another method of proof to show that if if $X \sim Poisson(\lambda)$ and $Y \sim Poisson(\mu)$, and $\lambda \geq \mu$, then X is stochastically greater than Y.

By definition, for integer $t \geq 0$, explicitly recognising the dependence on λ, we have

$$F_X(t;\lambda) = \sum_{x=0}^{t} \frac{e^{-\lambda}\lambda^x}{x!}$$

Consider the derivative of $F_X(t;\lambda)$ with respect to λ:

$$\begin{aligned}
\frac{dF_X(t;\lambda)}{d\lambda} &= \sum_{x=0}^{t}\left[-\frac{e^{-\lambda}\lambda^x}{x!} + \frac{xe^{-\lambda}\lambda^{x-1}}{x!}\right] \\
&= -\sum_{x=0}^{t}\frac{e^{-\lambda}\lambda^x}{x!} + \sum_{x=1}^{t}\frac{e^{-\lambda}\lambda^{x-1}}{(x-1)!} \\
&= -\sum_{x=0}^{t}\frac{e^{-\lambda}\lambda^x}{x!} + \sum_{x=0}^{t-1}\frac{e^{-\lambda}\lambda^x}{x!} \\
&= -\frac{e^{-\lambda}\lambda^t}{t!} \\
&= -P[X = t] < 0.
\end{aligned}$$

Thus, as the derivative is negative for all t whatever the value of λ, $F_X(t;\lambda)$ is a decreasing function of λ. Hence, if $\lambda \geq \mu$, then

$$F_X(t;\lambda) \leq F_X(t;\mu) = F_Y(t;\mu)$$

for all integers $t \geq 0$. As X and Y have discrete distributions on the same support, this relationship holds for all $t \in \mathbb{R}$.